Manual of Weighing Applications

Part 2

Counting

Preface

In many everyday areas of operation, the scale or the weight is only a means to an end: the quantity that is actually of interest is first calculated from the weight or the mass. Therefore, this weighing applications handbook covers the most important applications in individual parts, each of which is dedicated to a particular topic.

Counting with the help of scales is the subject of Part 2 of the handbook, which was recently completed. The most important question that arises from the counting application undoubtedly concerns the attainable degree of accuracy – or, conversely, the quantity of the unavoidable counting error.

Four tables in the chapter entitled "Selecting the "Right" Counting Scale" (beginning on p. 20) provide a quick overview of the counting error that can be reached (or falls short of being reached) under certain conditions.

The error can be precisely calculated for each individual case using a simple spreadsheet in the EXCEL file "ACCURACY.XLS," which is included with this handbook.

The fundamentals for determining piece counts based on weights are explained in the handbook. In addition, it illustrates which quantities determine the counting error and how. Some statistical fundamentals, which provide a better understanding of the counting procedure and of error calculation, are presented at the beginning of the counting applications handbook.

The goal of this handbook is to provide sartorius employees and interested clients with a comprehensive compilation of information related to the most important weighing applications – for use both as a training guide in a particular subject area and as a reference. In addition, it will supplement or update the user's knowledge of the subject.

Experiences from cooperation with users in laboratories and industry should give the weighing applications handbook an "interactive" feeling and enable its continued development through user input – in the form of short application reports, among other things. With time, these reports will supplement the handbook and make new and interesting applications available to everyone.

Marketing, Weighing Instruments
October 1999

Meaning of Symbols

Indices	
ref	related to the reference number or reference weight
x	quantity sought
(1)	related to 1 (representative) part
i	general notation for consecutive numbering of individual values
1, 2, ...	consecutive numbering of individual measured values or parts

Symbol	Quantity
W	weight
$\overline{W}$ pronounced: W bar	mean value of the weights
x	any measured value
$\overline{x}$ pronounced: x bar	mean value
Δx	scatter of the individual values
$\Delta\overline{x}$	scatter of the mean value
n	number of measurements or individual parts
s	standard deviation
s^2	variance
$\frac{s}{\overline{x}}$	coefficient of variation, relative standard deviation
t	statistical factor, STUDENT factor
P	confidence interval

Contents:

Counting – a Brief Overview

Information that is actually of interest to the user is often first calculated from values that were measured using scales. Counting is a weighing application that is used in all areas that depend on quick and reliable determination of the number of individual pieces in a large quantity of equal parts.

The principle behind counting is very simple: the mean value of the individual average piece weight is calculated based on the weight of a small number of individual parts (the reference piece count). The weight of a large quantity of the same individual parts (the total weight) then only needs to be measured and divided by this mean value to determine the piece count of the weighed quantity. Today, software installed on scales automatically converts the weight into a piece count. The user only needs to perform two weighing operations and enter the number of reference samples into the scale's program.

The most important question that arises when counting by weighing relates to the reliability of the piece count result: " Are there really 1,000 screws on the scale as shown in the display, or could it be only 998, or maybe even 1,005?"

It isn't necessary to recount 1,000 (or ???) parts to obtain the answer; the error of the counting result can be calculated.

The counting error is influenced by numerous quantities, and the relationships are complex.

- The total error is influenced to the greatest extent by the uniformity of the weights of the individual parts. This means that the smaller the ever-present (minimal) weight differences are from piece to piece, the "more accurate" is the resulting total piece count calculated by the scale. This statistical variance of the average piece weight is described mathematically by the standard deviation of the individual parts. The differences in the weights of the individual parts normally result from manufacture and cannot be readily influenced. The attainable degree of accuracy is basically limited by the variance of the individual average piece weights – independent of the "counting scale" used.
- Of course, the total error of the counting result is influenced by the type of scale selected – the resolution of the scale used to determine the reference weight is of crucial significance here. Comparatively, the resolution of the scale is of lesser importance when determining the total weight or the total piece count. The resolution of the scale – or the smallest display digit – must always be selected in suitable proportion to the individual piece weight.
- The counting error can be minimized by using larger reference piece counts; for this purpose, Sartorius scales feature a so-called reference optimization function. By gradually increasing the reference piece count in defined increments, this feature optimizes the basis of calculation used to determine the piece count so that it closely approximates the actual conditions of the weight variance of the individual parts.

Determining large piece counts using scales is a quick and easy counting method. Selecting a suitable scale and using an appropriate piece count leads to optimum results – on the basis of the limits set by the variances of the weights of the individual parts.

General Fundamentals

Examples for Use of "Counting Scales"

Counting is the most frequently used weighing application today. It is used extensively in the most diverse areas, such as:

- incoming inspection
- manufacturing
- warehousing
- packing, and
- shipping.

Counting also finds usage in all industrial branches, such as:

- precision engineering
- the electronics and electrotechnical industries
- the plastics industry
- paper manufacturing
- button manufacturing
- in print shops and in book and magazine publishing houses
- … .

Determining the Piece Count Based on the Weight

The basic principle behind counting using a scale or by weighing is very simple: if the parts to be counted all have the same weight, the number of individual parts , can be determined by dividing the combined weight of all parts by the weight of an individual part:

$$\text{piece count} = \frac{\text{weight of all parts}}{\text{weight of an individual part}}$$

or

$$n_x = \frac{W_x}{\overline{W}_{(1)}} . \qquad (1)$$

In the process, the average weight of an individual part is first determined by weighing a small, hand-counted number of individual parts – the so-called reference sample quantity:

$$\overline{W}_{(1)} = \frac{W_{ref}}{n_{ref}} . \qquad (2)$$

The equation used for counting with the scale – depending on the weight of the parts to be counted W_x (also referred to as the total weight), the weight of the reference samples W_{ref} and the number of reference samples n_{ref} – is as follows:

$$\boxed{n_x = W_x \cdot \frac{n_{ref}}{W_{ref}}} \qquad (3)$$

In reality, the mass of an individual part varies from piece to piece. Therefore, the determination of the piece count is subject to a statistical error. This means that once a certain number of parts is reached, the range of error for counting by weighing becomes so large that the accuracy of the piece count down to one part can no longer be ensured. If the calculated piece count deviates by exactly ± 0.5 from a whole number (e.g., 100.5), it is uncertain whether the piece count is actually 100 or 101.

The more the individual weights of the parts to be counted vary from their mean value, the larger is the error that can occur while determining the piece count.

The following sections will briefly illustrate the fundamentals of statistics – as far as they are necessary for understanding counting accuracy. Examples will be given and figures will be presented for calculating the counting error under various conditions.

Finally, systematic errors that can occur while determining the piece count based on the weights of the parts will be explained. These errors are often greater than statistical or random errors but can largely be avoided by adhering to standard operating procedures (see p. 24, Sampling while determining the reference weight), while, in general, statistical errors always occur.

Fundamentals of Statistics

In statistics, inferences as to the population are made on the basis of information gathered from small, random samples with the help of applicable mathematical models. The most important quantities for characterizing, for example, the uniformity of the weight (mass) of “identical” parts are the mean value and the standard deviation.

Mean Value

The mean value $\overline{W}$ (read as “W bar”) of a larger number of “equal” parts is calculated from the sum of the individual weights W_1 through W_n, divided by the number n of individual weights. Generally speaking, the mean value $\bar{x}$ of a series of weight measurements is expressed as follows:

$$\bar{x} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i \; . \qquad (4)$$

or, applied to the example below where six (n = 6) individual weighing operations W_1 through W_6 were performed:

$$\overline{W} = \frac{W_1 + W_2 + W_3 + W_4 + W_5 + W_6}{6} \qquad (5)$$

Two different series of weight measurements with “the same” mean value should be considered:

Weight	Weighing Series 1	Weighing Series 2
1	6 g	9.999 g
2	8 g	9.998 g
3	14 g	10.002 g
4	12 g	10.000 g
5	15 g	10.001 g
6	5 g	10.000 g

In the example above, the mean value of both weighing series is:

	Weighing Series 1	Weighing Series 2
Number of individual measurements n	6	6
Sum of the individual results	60 g	60.000 g
Mean value $\overline{W}$	10 g	10.000 g

By itself, the mean value derived from a series of individual weight measurements provides no information about the accuracy, reproducibility, scatter, or reliability of the measurements or of the parts being weighed. The greater the number of individual weight measurements n is, the better the mean value of these few weights describes the “true value” of all underlying weights.

When applied to the task of counting, this means that the mean value calculated while determining the average piece weight of a large number of individual parts – a large reference piece count – is “more accurate.” In other words, the total distribution is described more precisely and, therefore, the counting result becomes more exact.

The Meaning of Standard Deviation and Confidence Interval

The standard deviation is the quantity that provides the most important information for statistical evaluation of a series of weight measurements. The scatter of the individual values is calculated from the standard deviation with the help of statistical factors, e.g., for a specified confidence interval of 99 %.

The standard deviation is calculated according to this general formula[1] :

$$s = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2} \quad . \qquad (6)$$

or – applied to the example where n = 6

$$s = \sqrt{\frac{(W_1 - \overline{W})^2 + (W_2 - \overline{W})^2 + (W_3 - \overline{W})^2 + (W_4 - \overline{W})^2 + (W_5 - \overline{W})^2 + (W_6 - \overline{W})^2}{6 \cdot 1}} \qquad (7)$$

[1] Function for calculating the standard deviation according to equation (6) in EXCEL: STABW(...)

The **relative standard deviation**, i.e., the standard deviation related to the mean value $\frac{s}{\overline{W}}$ of the series of weight measurements, is often indicated in percent.

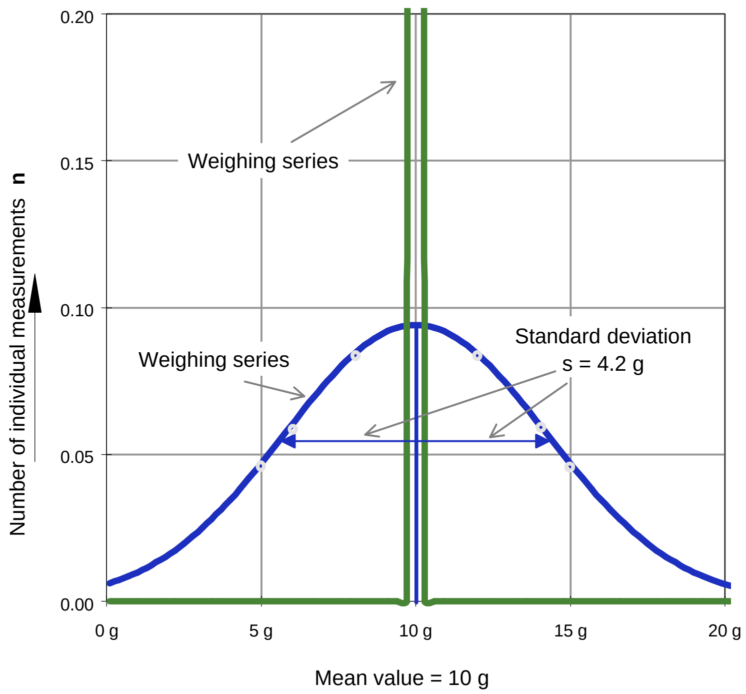
In some cases, references are made to the **variance**, which is indicated by the value s^2. However, this value has no other meaning than the standard deviation s itself.

(Sometimes, for example in OIML[2] R 76, an approximation of the standard deviation is also given for a series consisting of <u>6 individual weight measurements</u>: the difference between the maximum value and the minimum value is divided by 3: $s = \frac{x_{max} - x_{min}}{3}$, when n = 6.)

	Weighing Series 1	Weighing Series 2
Standard deviation s	4.24 g	0.00141 g
Relative standard dev. $\frac{s}{\overline{W}}$ in %	42 %	0.014 %
Approximation of the standard deviation in %	33 %	0.013 %

The following figure shows both of the weighing series referred to in the example. These two weighing series are so different – despite their “shared” mean value of 10 g or 10.000 g – that they cannot be presented in one graph. The six individual weights are marked on the curve by circles. From these few individual weights, an infinitely large number of values is inferred, which are presented in a bell curve.

[2] Organisation Internationale de Métrologie Légale



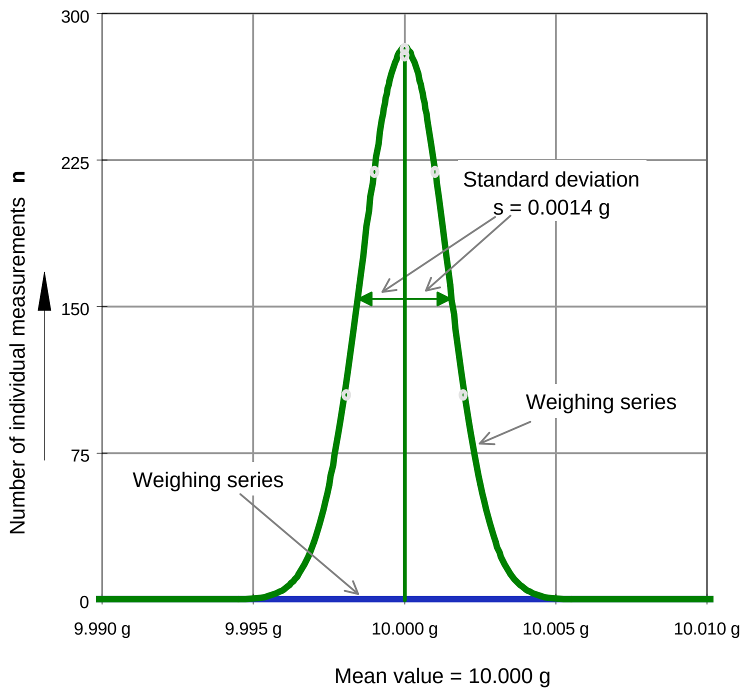


Figure 1: Distribution curves for the weighing series referred to in the example – for weighing series 1, only a few of the individual weights fall within a broad range to the right and left of the mean value. For weighing series 2, almost all individual weights lie within an extremely narrow range around the mean value $\overline{W}$.

To begin with, the distribution curve (see Figure 2) should be viewed in a general sense. If a particular quantity x is repeatedly measured under identical conditions, the measured values fall within in a certain range, and the value that is measured most frequently lies about in the middle of this range – as long as only random errors occur. Most measured values deviate only slightly from the value most frequently measured, and large deviations from the middle of the range are rare. If the frequency n, with which the individual measured values occur, is plotted against the measured value x, a distribution curve results. If the number of measurements is very large, this distribution curve turns into a bell curve – the Gaussian distribution curve .

The maximum value of this curve, the value most frequently measured, is the most probable value and corresponds to the mathematical mean value $\bar{x}$. The mean value derived from a few individual values is not identical to the true value, although it approximates the true value as the number of measurements n increases.

The standard deviation times 2 ($2 \cdot s$) is equal to the width of the bell curve at its points of inflection. Within this range of $\bar{x} \pm s$, 68.3 % of the individual values are found. In other words, the individual values lie within the range of $\bar{x} \pm s$ with a confidence interval of P = 68.3 %.

Within the range of the standard deviation times two or times three around the mean value ($\bar{x} \pm 2s$ or $\bar{x} \pm 3s$), 95.4 % or 99.73 % of all values of the distribution are found.

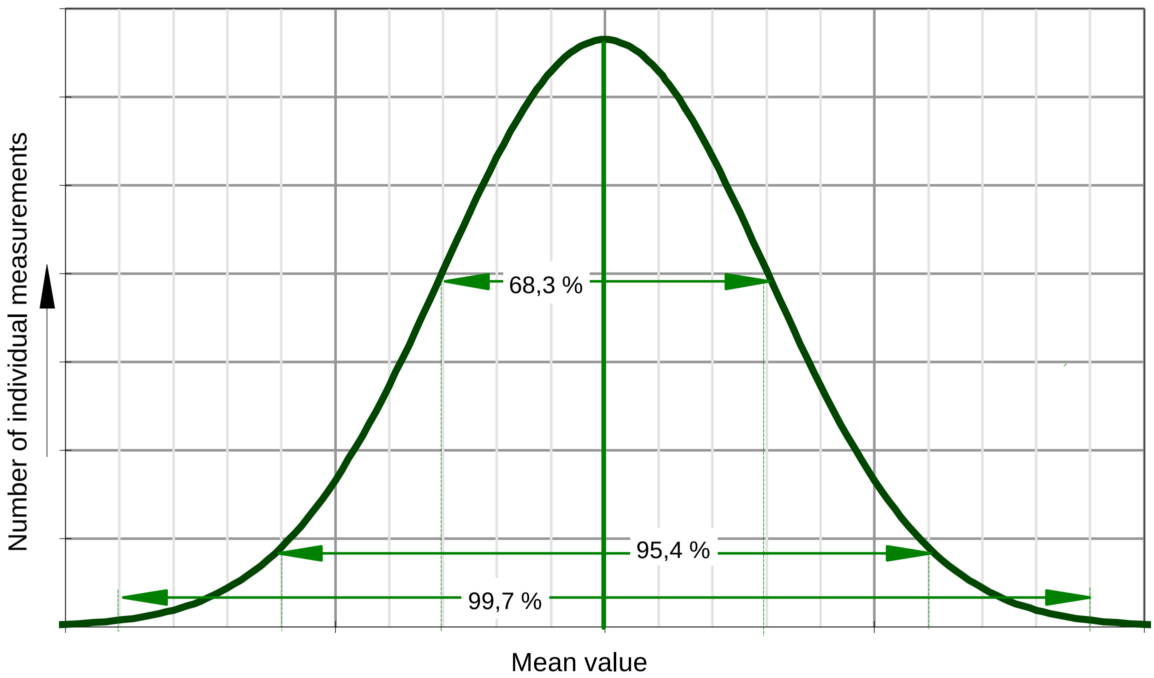


Figure 2: Gaussian Distribution

A confidence interval of, for example, P = 95 % means that the respective information is true in 95 out of 100 cases, or that 95 of 100 repeat measurements lie within the indicated range – or that the probability of the particular information being false is 5 %.

Both weighing series presented above, which each consist of 6 individual weight measurements with a mean value of 10 g or 10.000 g, can now be evaluated, e.g., with a confidence interval of 95 %:

The scatter of the individual values Δx is calculated by multiplying the standard deviation by a factor: namely, the t factor or student factor (this factor can be found in statistical tables; see also p. 27 of the appendix). The factor t depends on the required or desired confidence interval P and the number n of measured values. All individual values of the distribution, which is based on the observed series of weight measurements, lie within the boundaries of $\bar{x}+\Delta x$ and $\bar{x}-\Delta x$ with the selected confidence interval.

The scatter Δx is calculated as follows:

$$\Delta x = t \cdot s \qquad (8)$$

When n = 6 and P = 95 %, t = 2.57. The scatter can thus be calculated for both examples.

	Weighing Series 1	Weighing Series 2
$\bar{x}+s$	14.2 g	10.001 g
$\bar{x}-s$	5.8 g	9,999 g
Scatter $\Delta x = t \cdot s$	2.57· 4.243 g = 10,9 g	2.57·0.00141 g = 0.00363 g
$\bar{x}+\Delta x$	10 g + 10.9 g = 20.9 g	10.000 g + 0.004 g = 10.004 g
$\bar{x}-\Delta x$	10 g - 10.9 g (= - 0.9 g)	10.000 g - 0.004 g = 9.996 g

If one interprets the results, it follows that:

- With a confidence interval of 68.3 % (strictly speaking, related to infinitely many measured values), the individual weights of weighing series 1 lie in the range of 5.8 g to 14.2 g; those of weighing series 2 are in the range of 10.001 g to 9.999 g.
- With a confidence interval of 95 % when n = 6 weights, the scatter calculates to 10.9 g or 10.004 g for weighing series 1 or 2.
- The individual weights of weighing series 1, therefore, fall between 0 and 20.9†g with a confidence interval of 95 % and a mean value of 10 g.
- The individual weights of weighing series 2 fall between 9.996 g and 10.004 g, with a confidence interval of 95% and a mean value of 10.000 g. 5 % of the individual weights lie outside of this range of 9.996 g to 10.004 g.

Calculating the Counting Error

Piece Count Error

The most important question that arises when counting by weighing undoubtedly concerns the attainable degree of accuracy – or the quantity of the unavoidable counting error.

Taking into account the error of the scale or scales and the variance of the weights of the individual parts, application of the general rules of error calculation yields the expression for the standard deviation of the piece count shown in equation (9):

This seemingly obscure expression includes all random errors (no systematic errors, see p. 24) that influence the accuracy of the counting result. All together, there are six different quantities:

- the number of parts to be counted n_x
- the reference piece count n_{ref}
- the average individual weight of a part $W_{(1)}$
- the variance of the weight of the individual parts s_{Wref}
- the variance of the weight of the reference sample quantity $s_{W(1)}$
- the variance of the weight of the total piece count s_W

On condition that the reference piece count n_{ref} was determined error-free, resulting errors can basically be attributed to two different influences:

- the influence of the weighing instrument, i.e., the scale and
- the influence of the statistical variance of the weight of the individual parts.

Of course, both of these fundamental causes of error apply to the determination of the reference weight W_{ref} as well as to the determination of the total weight W_x.

Once the transition is made to the standard deviation of the reference weight or that of the total weight, the equation for calculating the standard deviation s_x of the parts to be counted n_x ultimately results:

$$s_x = \sqrt{\frac{n_x^2}{n_{ref}^2} \cdot \underbrace{\left(\frac{s_{W_{ref}}}{W_{(1)}}\right)^2}_{\text{Reference weight error}} + \underbrace{\left(\frac{s_{W_x}}{W_{(1)}}\right)^2}_{\text{Total weight error}} + \left(\frac{n_x^2}{n_{ref}} + n_x\right) \cdot \underbrace{\left(\frac{s_{W_{(1)}}}{W_{(1)}}\right)^2}_{\text{Individual piece variance}}} \qquad (9)$$

Comparison of Various Influence Quantities on Counting Accuracy

Different quantities influence the end result to varying degrees:

$$s_x^2 = \frac{n_x^2}{n_{ref}^2} \cdot \left(\begin{array}{c}\text{Variance of the}\\ \text{reference weight}\end{array}\right) + 1 \cdot \left(\begin{array}{c}\text{Variance of the}\\ \text{total weight}\end{array}\right) + \left(\frac{n_x^2}{n_{ref}} + n_x\right) \cdot \left(\begin{array}{c}\text{Individual piece}\\ \text{variance}\end{array}\right)$$

If one considers the three different partial errors which, as a sum, yield the standard deviation of the total piece count, it appears that

- the variance of the total weight is multiplied by the factor 1
- the variance of the reference weight is multiplied by the factor $\left(\frac{n_x^2}{n_{ref}^2}\right)$ and
- the variance of the individual average piece weight is multiplied by the factor $\left(\frac{n_x^2}{n_{ref}} + n_x\right)$.

The table below includes some counting examples for various values of n_x and n_{ref}. Like the graph which follows (Figure 3), these examples show that the influence of the individual piece variance can easily exceed that of the reference weight variance by several decimal powers.

		Varianceof the total weight multiplied by	Variance of the reference weight multiplied by	Individual piece variance multiplied by
n_x	n_{ref}		n_x^2/n_{ref}^2	$(n_x^2/n_{ref})+n_x$
100	10	1	100	1.100
100	50	1	4	300
1.000	10	1	10.000	101.000
1.000	50	1	400	21.000
1.000	100	1	100	11.000
10.000	10	1	1.000.000	10.010.000
10.000	50	1	40.000	2.010.000
10.000	100	1	10.000	1.010.000
		…equals s_x^2 when the three components are		

Table 1: Order of magnitude for factors with which the particular variance must be multiplied before the three components comprising the total variance of the counting result are summed.

The factor $\left(\frac{n_x^2}{n_{ref}} + n_x\right)$ is always greater than $\left(\frac{n_x^2}{n_{ref}^2}\right)$, and $\left(\frac{n_x^2}{n_{ref}^2}\right)$ is always greater than 1; this follows from the numbers in Table 1 but also results based on the following consideration:

If the reference piece count n_{ref} becomes very large compared to the piece count n_x (or eventually = n_x), the factor $\frac{n_x^2}{n_{ref}^2}$ approaches 1. Normally, $n_x >> n_{ref}$ and, therefore, $\frac{n_x^2}{n_{ref}^2} >> 1$.

Because $n_{ref}^2 > n_{ref}$, it follows that $\frac{1}{n_{ref}^2} < \frac{1}{n_{ref}}$ and, consequently, $\left(\frac{n_x^2}{n_{ref}^2}\right) < \left(\frac{n_x^2}{n_{ref}} + n_x\right)$. It can, therefore, be shown that the variance of the weight of the individual parts has the greatest influence on the accuracy of the counting result.

In order to provide an at-a-glance overview of the influence exerted by these various sources of error on the total result, the following graph is prepared. It shows the reference piece count on the x axis. The y axis indicates by which factor the influence of the variance of the individual parts and of the reference weight is larger than that of the variance of the total weight. These values are plotted as a function of the reference piece count. The graph (Figure 3) is valid for quantities of 1,000 parts.

The curves show that the counting accuracy is determined to the greatest extent by the variance of the individual weights around their mean value. This influence is greater than that of the reference weight variance and the variance of the total weight by several decimal powers. For a piece count of 1,000, the influence of the individual piece variance on the variance of the counting result is approximately 10,000 times greater than that of the variance of the total weight and still 100 times greater than the influence of the reference weight variance.

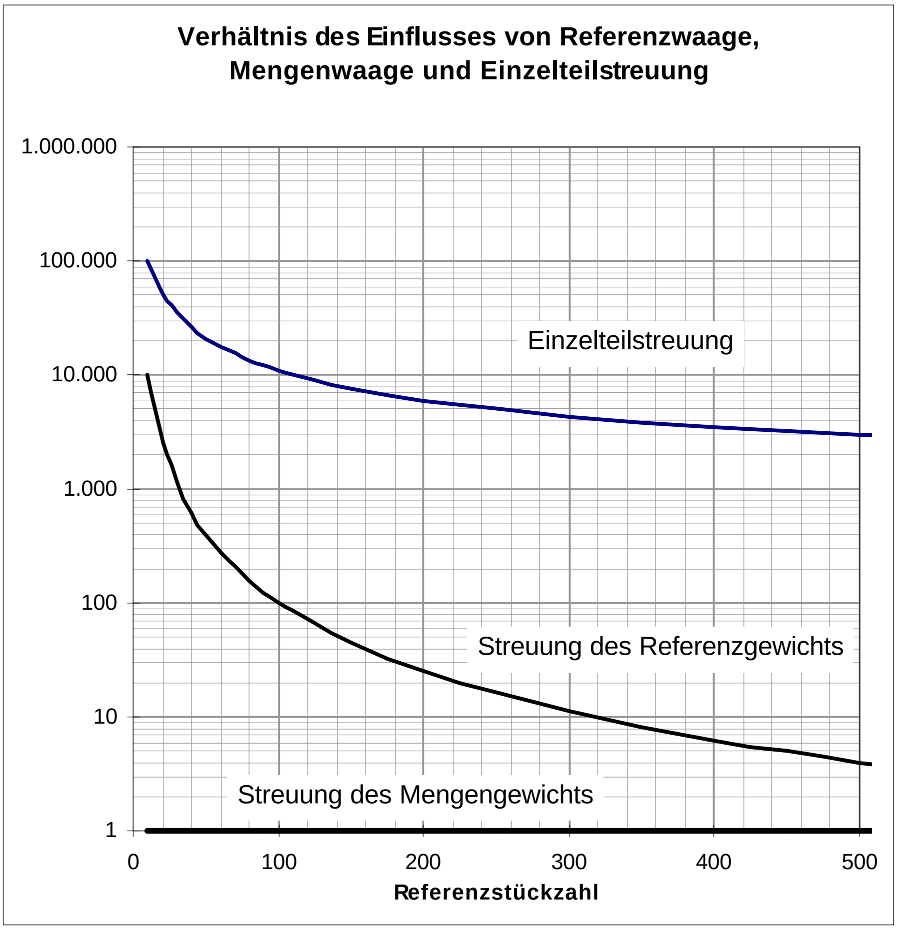


Figure 3: Relation of the influence on the total counting error when 1,000 parts are to be counted – the numbers on the y axis indicate by which factor the variance of the total weight, the reference weight and the individual average piece weight is calculated to determine the total error.

Determining the Counting Accuracy

The standard deviation s_x of the piece count n_x can be calculated according to the relationship shown in equation (9). By multiplying the result by the t factor, one can then determine with the desired confidence interval the maximum deviation Δx from the target piece count for the counting task.

$$\Delta x = t \cdot \sqrt{\frac{n_x^2}{n_{ref}^2} \cdot \left(\frac{s_{W_{ref}}}{W_{(1)}}\right)^2 + \left(\frac{s_{W_x}}{W_{(1)}}\right)^2 + \left(\frac{n_x^2}{n_{ref}} + n_x\right) \cdot \left(\frac{s_{W_{(1)}}}{W_{(1)}}\right)^2}$$

$$\Delta x = t \cdot s_x \qquad (10)$$

If the relative error should be indicated in %, Δx must be related to the total piece count n_x:

$$\Delta x \text{ (in \%)} = \frac{t \cdot s_x}{n_x} \cdot 100$$

The counting accuracy for certain representative conditions has been calculated and presented in graphs, see appendix, Figure 9 through 20.

The result is always plotted on the y axis. It corresponds to the standard deviation s_x related to the target piece count n_x. Curves are shown for a confidence interval of 99% (dotted line) and 99.9 % (solid line), respectively.

The following were depicted in two different ways (i.e., used as independent quantities and plotted on the x axis of the graphs):

- the standard deviation of the individual weights related to the mean value of the individual average piece weight $\frac{s_{W_{(1)}}}{W_{(1)}}$.
- the reference piece count n_{ref} (from 0 to 500 on the upper curve, from 0 to 100 on the lower curve, respectively)

Quantities assumed to be constant for the respective curve and the corresponding numerical values are indicated in the graphs. The curves are calculated for n_x = 10 000; with smaller piece counts n_x , the curves move upward on the graph, i.e., the counting error becomes larger (see Figure 7 and Figure 8).

The graphs represent values of $W_{(1)}$ = 1 digit[3], $W_{(1)}$ = 10 digits and $W_{(1)}$ = 100 digits. The average weight of the individual parts as it relates to the readability of the scale determines which graph should be used to determine the counting error.

For the standard deviations $s_{W_{ref}}$ and s_{W_x} according to equation 9 (p. 11) or equation 10, 1 digit (or 10 digits or 100 digits) is calculated with the value. In other words, it is assumed that the resolution of the scale used is the same for determining both the reference and the weight of the total number of parts (see Figures 9 and 15, 10 and 16, 13 and 19).

[3] For instruments with a digital display, this is the smallest digital step.

For the graphs (Figures 11, 14, 17, and 20), a scale with an internal resolution 10 times higher than the display resolution was used to determine the reference weight, i.e., $s_{W_{ref}}$ is calculated with 0.1 digit. The standard deviation of the total weight s_{W_x} continues to be calculated with 1 digit.

The same holds true for the next graphs (12 and 18): here, a scale with an internal resolution 100 times higher than the display resolution was used to determine the reference weight [4]. In other words, $s_{W_{ref}}$ is calculated with 0.01 digit to determine the counting accuracy, and s_{W_x} is calculated with 1 digit.

The graphs provide a good orientation guide for many counting applications. To enable the counting accuracy to be calculated for each individual case, the EXCEL file "ACCURACY" is included with this handbook. After 10 individual average piece weights, the readability of the scale, and the target piece count and reference piece count are entered in the spreadsheet, the relative counting error is calculated with a confidence interval of 95%, 99% and 99.9%. In addition, the maximum deviation from the target piece count is indicated, and a determination is made as to whether or not counting that is accurate down to the last component is possible under the selected conditions. Examples of printouts are included in the appendix (p. 43).

If the absolute error when counting may not be larger than 1 piece, the condition

$$\Delta x = t \cdot s_x < 0{,}5$$

must be fulfilled.

[4] The same is true if a separate reference scale with a readability of 1 / 100 of the bulk weigher is used.

Reference Sample Updating for Optimization

As explained in the previous chapter on calculating the counting error, one can generally improve the counting accuracy with larger reference piece counts.

Because it is very tedious to count a large number of mostly small, individual parts by hand, and, above all, because a (systematic) counting error, which would distort the entire counting result, can easily result, all Sartorius scales equipped with counting application software feature a reference sample updating option.

Starting out with a small reference piece count, this feature gradually increases it until the desired reference piece count has been reached. Certain conditions must be observed in the process to ensure that the counting error when determining the reference piece count is always less than 1 piece ($< \pm 0.5$).

$$n_{1ref} + 2 \leq n_{2ref} \leq 2 \cdot n_{1ref} \qquad (11)$$

For example, in the first step, the reference weight W_{1ref} is determined based on $n_{1ref} = 10$ hand-counted, individual parts. After adding 2 to 20 additional, individual parts, the piece count n_{2ref} is calculated based on the weight W_{2ref} according to the relationship (3) on page 4:

$$n_{2ref} = W_{2ref} \cdot \frac{n_{1ref}}{W_{1ref}} . \qquad (12)$$

Software for Sartorius scales allows reference sample updating for optimization

- if the current piece count is less than double the original piece count ($n_{2ref} \leq 2 \cdot n_{1ref}$),
- if the deviation from the next whole number of n_{2ref} is $< \pm 0.3$,
- and as long as the piece count n_{ref} is < 100.

If these conditions are met, the value $\frac{n_{1ref}}{W_{1ref}}$ for calculating the target piece count or for the next step in reference optimization is substituted by $\frac{n_{2ref}}{W_{2ref}}$. The resulting equation is:

$$n_{3ref} = W_{3ref} \cdot \frac{n_{2ref}}{W_{2ref}} \text{ or } n_x = W_x \cdot \frac{n_{2ref}}{W_{2ref}} . \qquad (13)$$

The following example, with the help of the accompanying graph, will illustrate and provide a better understanding of reference optimization. With the exception of the piece count n_x, the graph corresponds to Figure 16 on page 37. (Figures 15 through 20 can be used accordingly).

- the counting error should not exceed 0.5 %,
- fluctuations in the weights of the parts being counted amount to $\frac{s_{W1}}{W_{(1)}} = 1$ % when the standard deviation of $s_{W1} = 0.012$ g
- one digit on the scale corresponds to 0.1 g
- the average individual piece weight is $W_{(1)} = 1.2$ g, corresponding to ~ 10 digits

Since the predefined counting error tolerance may not exceed 0.5 %, a reference piece count of at least 80 results from the curve (with a confidence interval of 99.9 %).

In the first step of reference sample updating, a reference piece count of 10 is selected. The counting error for this piece count is ~ 3 %, i.e., 0.3 part for every 10 parts. This ensures accurate counting down to the last component, and the number of reference samples can, therefore, be raised by 10.

For a reference piece count of 20, the counting error is ~ 1.5 %, under the given conditions, i.e., 0.3 part for every 20 parts. Accurate counting down to the last component is again ensured.

If the piece count is raised to 40 in the next step, the counting error is still ~ 0.9 %, i.e., a maximum of 0.36 part for every 40 parts. The counting error remains less than 1 piece or < ± 0.5, and the next time the piece count is doubled, a reference piece count of 80 will already be reached. At this point, the actual counting task can begin with the desired degree of counting accuracy.

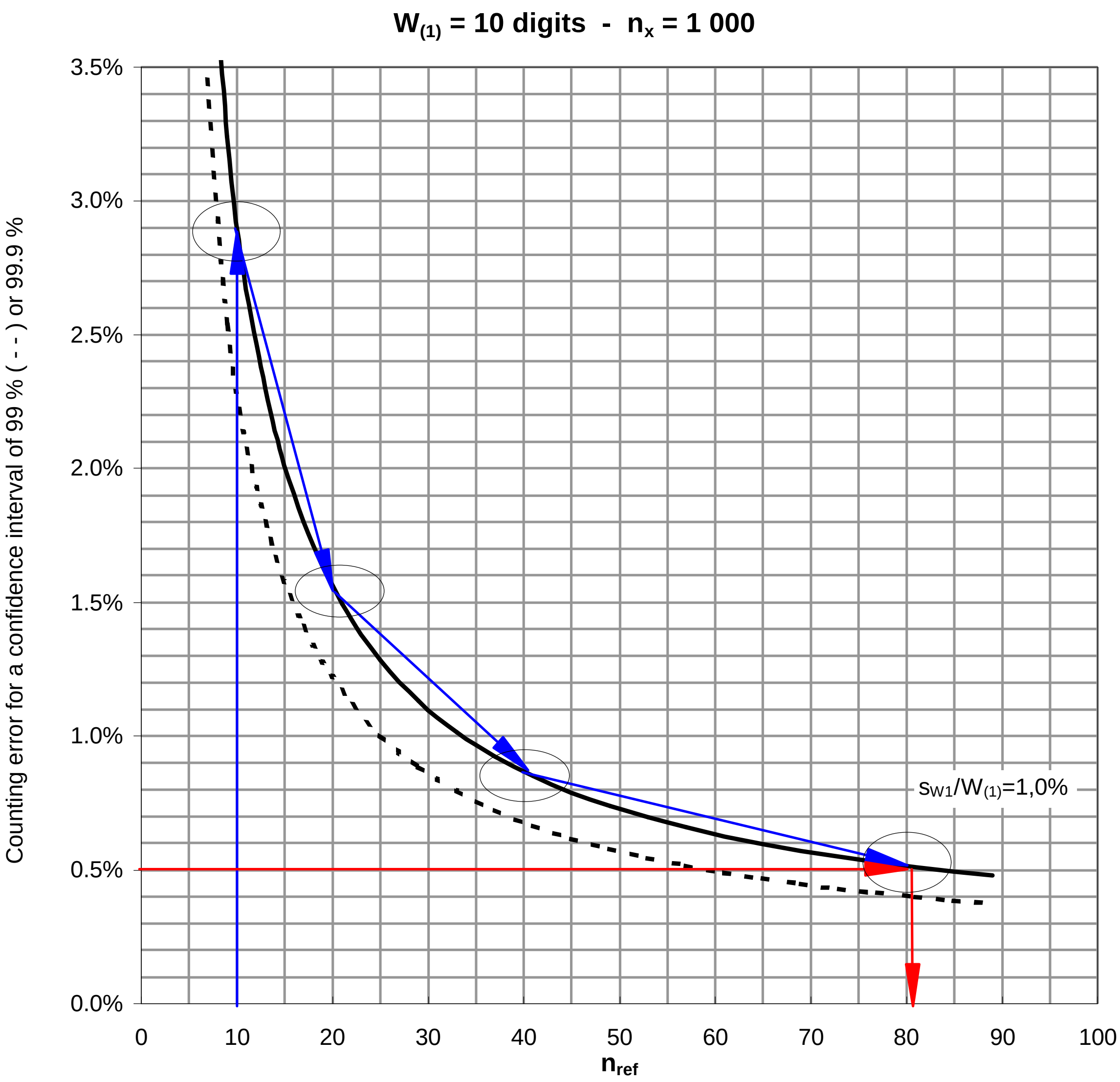


Figure 4: Illustration of reference sample updating

Selecting the "Right" Counting Scale

When selecting a counting scale, the general rules apply. Above all, the following should be taken into consideration:

- the expected maximum load or the total weight which will be placed on the scale
  (If necessary, the total weight can be divided into smaller amounts, which can then be added together in the totalizing memory of the scale.)
- the readability, which is determined by the desired accuracy of the counting result. This, in turn, depends on
  - the absolute quantity of the reference weight,
  - the accuracy with which the (average) individual piece weight can be read on the display, and
  - the accuracy with which the reference weight can be read on the display

In general, the following applies: For determining the reference, a higher internal resolution of the weighing range – or the use of a reference scale with a higher resolution than that of the bulk weigher – greatly improves the counting accuracy and, in addition, reduces the required reference piece count.

The four tables that follow should simplify the selection of a suitable scale. All values are calculated for a confidence interval of 99.9 %.

Determining the Suitable Readability of a Scale

First of all, a decision must be made about the degree of accuracy with which the parts are to be counted – this determines which table (counting error < 1 %, < 0.5 %, < 0.1 % or < 0.05 %) is applicable.

The next parameter that must be known is the relative standard deviation ($s / W_{(1)}$) of the parts to be counted – the tables include examples for four different values (5 %, 1 %, 0.5 % and 0.1).

The individual average piece weights are indicated in digits, i.e., an immediate reference is made regarding the readability of the scale. Refer to the lines in the table marked with O (10-times higher internal resolution) or OO (100-times higher internal resolution) when using a reference scale or a scale with a higher internal resolution to determine the reference.

A readability of 100 digits related to the average individual piece weight with 100-times higher internal resolution rather than only 10-times higher resolution no longer presents an appreciable advantage for determining the reference.

The last column in the tables shows the reference piece count at which the corresponding counting errors are reached or fall short of being reached. If the calculated reference piece count is > 500, the counting error entered will be considered "impossible" to attain. The smallest entry for the reference piece count is 10, even if the error is clearly smaller than 1 % and reference piece counts smaller than 10 are being calculated (e.g., at the bottom of Table 1, where the standard deviation of the individual parts is 0.1 %).

Example:

The desired counting error is ≤ 0.5 % – therefore, Table 3 should be selected.

The average weight of the individual parts is 1.3 g. This weight was calculated based on the weights of 10 parts as described in the section entitled “Fundamentals of Statistics” on page 5. (For information on sampling, see also chapter „Removal of Reference Samples“, page 24.)

The standard deviation of the individual average piece weight is 0.0125 g. Therefore, the relative standard deviation is: $\frac{s}{W_{(1)}} = \frac{0,0125g}{1,3g} = 0,0096 = 0,96\% \approx 1\%$

Thus in Table 3, the second section from the top is applicable: with a scale readability of 1 g (individual average piece weight =1.3 g ≈ 1 digit), a counting error < 0.5 % could not be attained with the parts in the example. For readabilities of 0.1 g or 0.01 g (individual average piece weight =1.3 g ≈ 10 digits or ≈ 100 digits), the desired counting error is attainable under various conditions listed below – with the reference piece counts indicated in column three:

- When the readability of the scale is 1 g, and the resolution is 10-times (or 100-times) higher, with at least 92 (or 45) reference samples the required counting error can be attained.
- A reference sample piece count of at least 92 is required when the readability of the scale is 0.1 g and the reference resolution = total weight resolution. A reference sample piece count of at least 45 is necessary for a scale with a 10-times higher internal resolution. However, a reference sample piece count of 44 is sufficient for a 100-times higher resolution to reduce the selected counting error to below 0.5 %.
- When the readability of the scale is 0.01 g, and the reference sample resolution = actual piece count resolution or 10-times to 100-times higher resolution, reference piece counts of at least 44 or 45 can be used to attain the counting error of 0.5 %. With the parts in the example scales with higher resolution no longer present an advantage for determining a “better“ counting result.

In all cases, n_{ref} is so high that the automatic reference optimization function should be used. With this feature, the required reference piece count is achieved quickly – in three or four steps. It also largely rules out a systematic counting error that could occur while counting the reference samples by hand. Such an error would render the entire counting result unusable.

Counting error < 1 % at a confidence interval of 99,9%		
Relative standard deviation of the parts to be counted $s/W_{(1)}$	Individual weight W(1) in digits	Minimum reference piece count nref
5%	1	500
	1 ○	285
	1 ○○	280
	10	283
	10 ○	280
	10 ○○	280
	100	280
	100 ○	280
	100 ○○	280
1%	1	336
	1 ○	39
	1 ○○	12
	10	40
	10 ○	12
	10 ○○	11
	100	12
	100 ○	11
	100 ○○	11
0.5%	1	332
	1 ○	35
	1 ○○	10
	10	35
	10 ○	10
	10 ○○	10
	100	10
	100 ○	10
	100 ○○	10
0.1%	1	330
	1 ○	33
	1 ○○	10
	10	33
	10 ○	10
	10 ○○	10
	100	10
	100 ○	10
	100 ○○	10

○: 10-times higher internal resolution

○○: 100-times higher internal resolution

Table 2: Selection of a suitable scale when the maximum accepted counting error is 1 % for various relative standard deviations of the individual parts.
(10,000 parts were used as a basis for calculation; i.e., for low piece counts such as 100 parts, the error can be up to ~ 0.5 % larger than indicated above. See also Figures 7 and 8.)

A counting error of ± 1 % corresponds to counting that is accurate down to the last component when the total number of parts counted is 50. When 1,000 parts are counted, the result can deviate from the target by up to ± 10 parts.

Counting error < 0,5 % at a confidence interval of 99,9%		
Relative standard deviation of the parts to be counted $s/W_{(1)}$	Individual weight W(1) in digits	Minimum reference piece count nref
5%	independent of the scale selected	not possible
1%	1	not possible
	1 ○	92
	1 ○○	45
	10	92
	10 ○	45
	10 ○○	44
	100	45
	100 ○	44
	100 ○○	44
0.5%	1	not possible
	1 ○	72
	1 ○○	15
	10	72
	10 ○	14
	10 ○○	11
	100	14
	100 ○	11
	100 ○○	11
0.1%	1	not possible
	1 ○	67
	1 ○○	10
	10	67
	10 ○	10
	10 ○○	10
	100	10
	100 ○	10
	100 ○○	10

○: 10-times higher internal resolution

○○: 100-times higher internal resolution

Table 3: Selection of a suitable scale when the maximum accepted counting error is 0,5 % for various relative standard deviations of the individual parts.
(10,000 parts were used as a basis for calculation; i.e., for low piece counts such as 100 parts, the error can be up to ~ 0.5 % larger than indicated above. See also Figures 7 and 8.)

A counting error of ± 0.5 % corresponds to counting that is accurate down to the last component when the total number of parts counted is 100. When 1,000 parts are counted, the result can deviate from the target by up to ± 5 parts.

Counting error < 0,1 % at a confidence interval of 99,9%		
Relative standard deviation of the parts to be counted $s/W_{(1)}$	Individual weight W(1) in digits	Minimum reference piece count nref
5%	independent of the scale selected	not possible
1%	independent of the scale selected	not possible
0.5%	1	not possible
	1 ○	not possible
	1 ○○	320
	10	not possible
	10 ○	283
	10 ○○	279
	100	283
	100 ○	279
	100 ○○	279
0.1%	1	not possible
	1 ○	355
	1 ○○	42
	10	336
	10 ○	39
	10 ○○	12
	100	39
	100 ○	12
	100 ○○	11

○: 10-times higher internal resolution
○○: 100-times higher internal resolution

Table 4: Selection of a suitable scale when the maximum accepted counting error is 0.1 % for various relative standard deviations of the individual parts.
(10,000 parts were used as a basis for calculation; i.e., for low piece counts such as 100 parts, the error can be up to ~ 0.5 % larger than indicated above. See also Figures 7 and 8.)

A counting error of ± 0.1 % corresponds to counting that is accurate down to the last component when the total number of parts counted is 500. When 1,000 parts are counted, the result can deviate from the target by up to ± 1 part.

Counting error < 0,05 % at a confidence interval of 99,9%		
Relative standard deviation of the parts to be counted $s/W_{(1)}$	Individual weight W(1) in digits	Minimum reference piece count nref
5%	independent of the scale selected	not possible
1%	independent of the scale selected	not possible
0.5%	independent of the scale selected	not possible
0.1%	1	not possible
	1 ○	not possible
	1 ○○	135
	10	not possible
	10 ○	92
	10 ○○	45
	100	92
	100 ○	45
	100 ○○	44

○: 10-times higher internal resolution

○○: 100-times higher internal resolution

Table 5: Selection of a suitable scale when the maximum accepted counting error is 0.05 % for various relative standard deviations of the individual parts.
(10,000 parts were used as a basis for calculation; i.e., for low piece counts such as 100 parts, the error can be up to ~ 0.5 % larger than indicated above. See also Figures 7 and 8.)

A counting error of ± 0,05 % corresponds to counting that is accurate down to the last component when the total number of parts counted is 1,000.

Removal of Reference Samples

When counting by weighing, a factor is calculated based on the number of reference samples and the reference weight. With the help of this factor, the piece count can be inferred from the total weight (see equation 3).

The reference samples represent only a small fraction of the total number of parts to be counted. It is basically assumed that the qualities of this small number of parts correspond to those of the total number of parts (that they paint a "representative picture of the population"). Only then can statistical formulas that are based on a Gaussian distribution curve be used. A corresponding standard operating procedure for sampling is of crucial significance for the usability of the counting result.

Random or statistical errors that can occur when determining the piece count are covered extensively in chapter "Calculating the Counting Error " page 11. The sampling error as a systematic error, however, can easily become much larger than the statistical error. This means that the reference samples must be carefully selected and counted by hand accordingly.

Theoretically, each individual part from the total quantity must have the same probability of being included in the reference sample.

In practice this means that, for example, several individual parts are removed from a large container at 10 different locations (e.g., from the top, the bottom, the front, ...). These parts are then mixed together well. Next, this group of parts is divided in half, in fourths, and then in eighths. Finally, one part for the reference sample is taken in succession from each of these small groups. Alternatively, one can take small samples at regular intervals while the small parts are being manufactured, mix these samples together, divide them into groups, and finally remove parts from each small group for the determination of the reference weight.

Difficulties can arise if, for example, small parts are being produced by two machines and the parts manufactured by machine 1 have a slightly different mean value than those produced by machine 2. These conditions are demonstrated in Figures 5 and 6.

In this case, one can either

- count parts only from machine 1 or machine 2, which assumes a smaller individual piece variation and, accordingly, a greater accuracy of the counting result,
- or mix the parts from machine 1 and machine 2 at a defined proportion for sampling both while determining the reference weight and while counting. However, one must expect a larger individual piece variation and, accordingly, a larger counting error.

The graph shows (see Figure 5) that the total distribution becomes increasingly broader as the distance between the mean values of the individual distributions increases. In the process, the standard deviation or the variance also continues to increase. How one actually works under such conditions depends mainly on the acceptable counting error. If the mean values of the individual distributions are too far apart, the cumulative distribution (see Figure 6) no longer results in a Gaussian distribution curve, and the basis for using the statistical calculations applied here no longer exists. This means that under such conditions, parts manufactured by different machines may not be counted together.

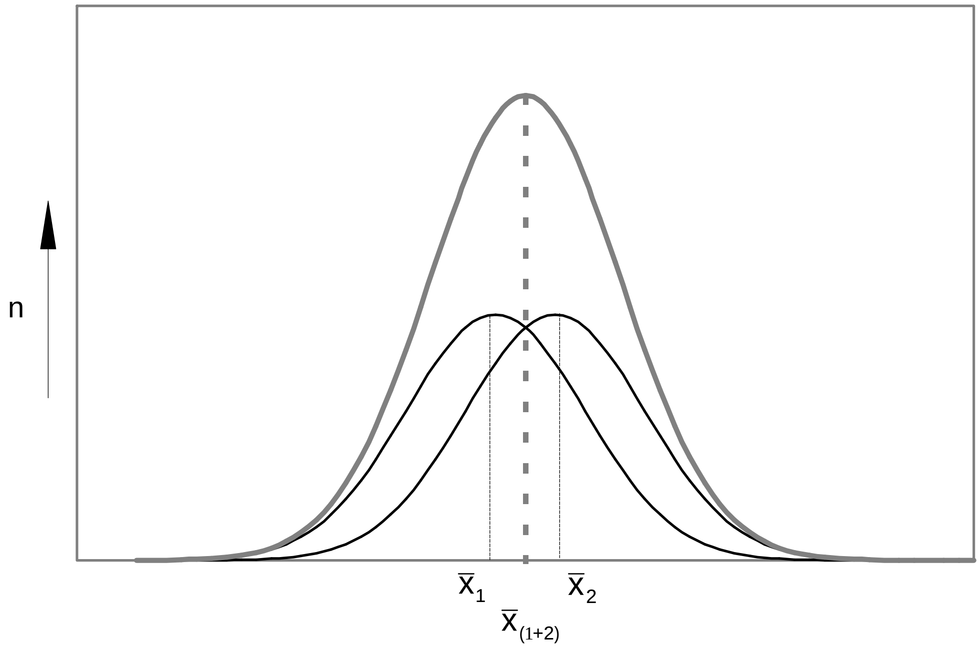


Figure 5: Frequency distribution of the parts manufactured by machine 1 with the mean value $\bar{x}_1$ and those produced by machine 2 with the mean value $\bar{x}_2$; frequency distribution for the combination of parts from machines 1 and 2 with the mean value $\bar{x}_{(1+2)}$

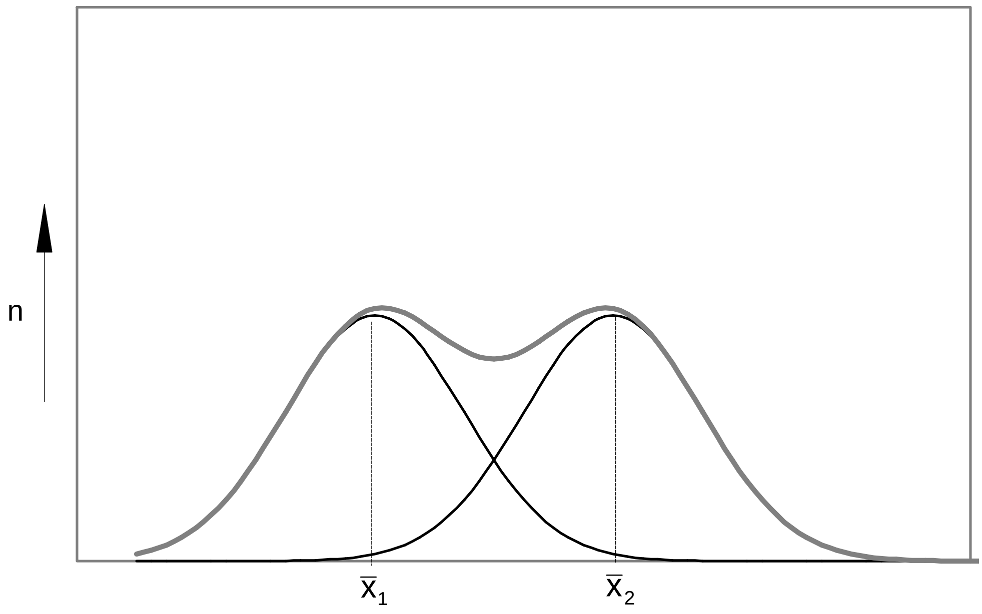


Figure 6: Frequency distribution of the parts manufactured by machine 1 with the mean value $\bar{x}_1$ and those produced by machine 2 with the mean value $\bar{x}_2$; a "bell curve" no longer results when the parts from machines 1 and 2 are added together, i.e., the parts no longer meet the fundamental requirement for the applicability of the statistical calculations.

Appendix

t Factor or Student Factor for Calculating the Ranges of Dispersion (Scatter)

To calculate the scatter $\Delta x = t \cdot s$ with a particular confidence interval, the t factors are required.

	t factor		
Confidence interval	95 %	99 %	99,9 %
n = 6	2,57	4,03	6,86
n = 10	2,26	3,25	4,781
n = 20	2,09	2,86	3,883
n = 50	2,009	2,678	3,469
n= 100	1,984	2,626	3,390
n = ∞	1,960	2,576	3,291

Graphs Determining the Counting Accuracy

Comments on the following graphs see page 14.

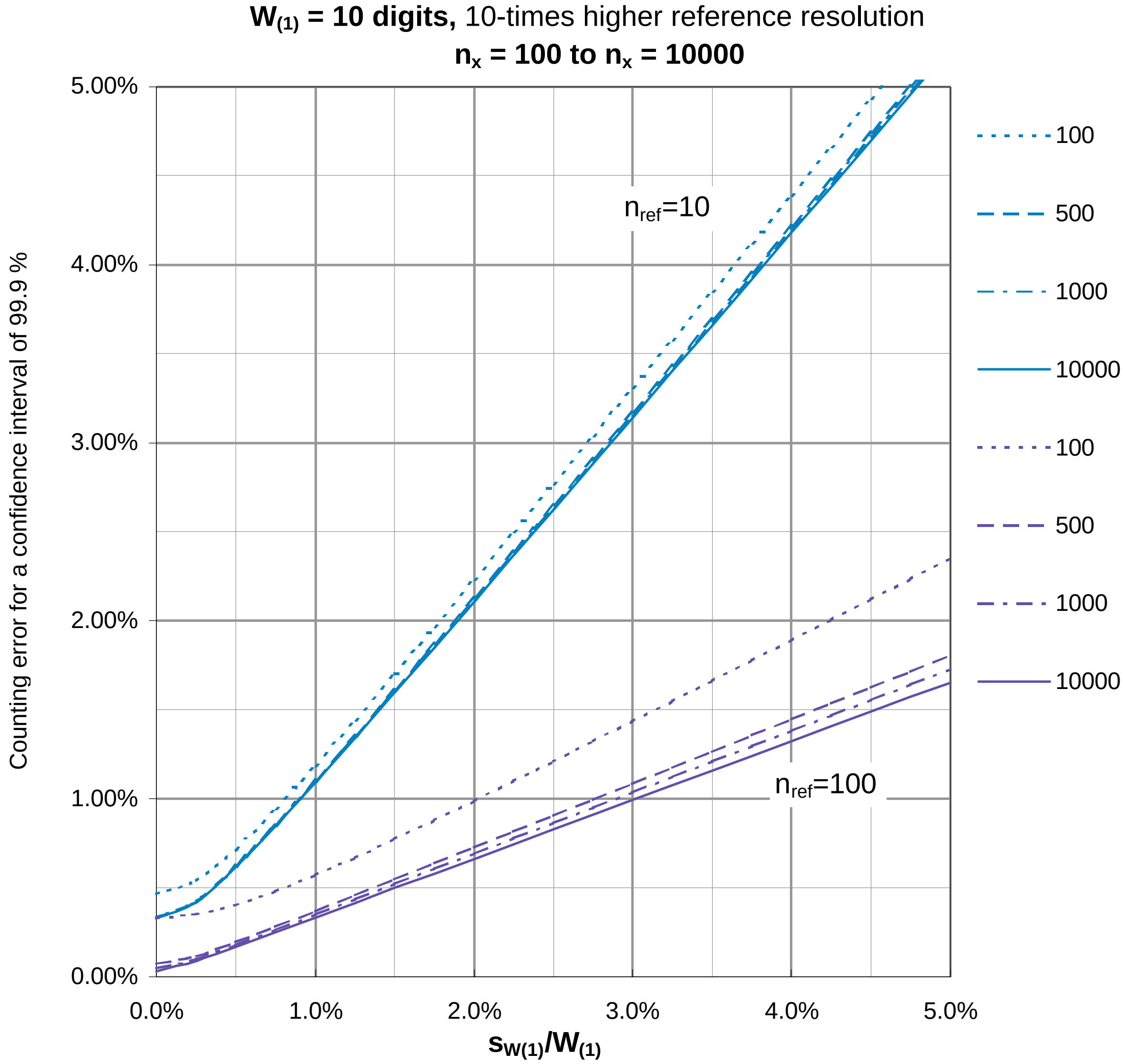


Figure 7: Relative counting error plotted as a function of the relative standard deviation of the individual parts for various target piece counts n_x.

The graph shows that the smaller the total number of parts to be counted is, the larger the relative counting error becomes. For example, for a relative standard deviation of 3.0% and a reference piece count of 10, the counting error equals

- 3.1 % for 10,000 parts
- 3.2 % for 1,000 parts
- and 3.3 % for 100 parts.

For a relative standard deviation of 3 % and a reference piece count of 100, the counting error corresponds to

- 1.0 % for 10,000 parts
- 1.1 % for 1,000 parts
- and 1.4 % for 100 parts.

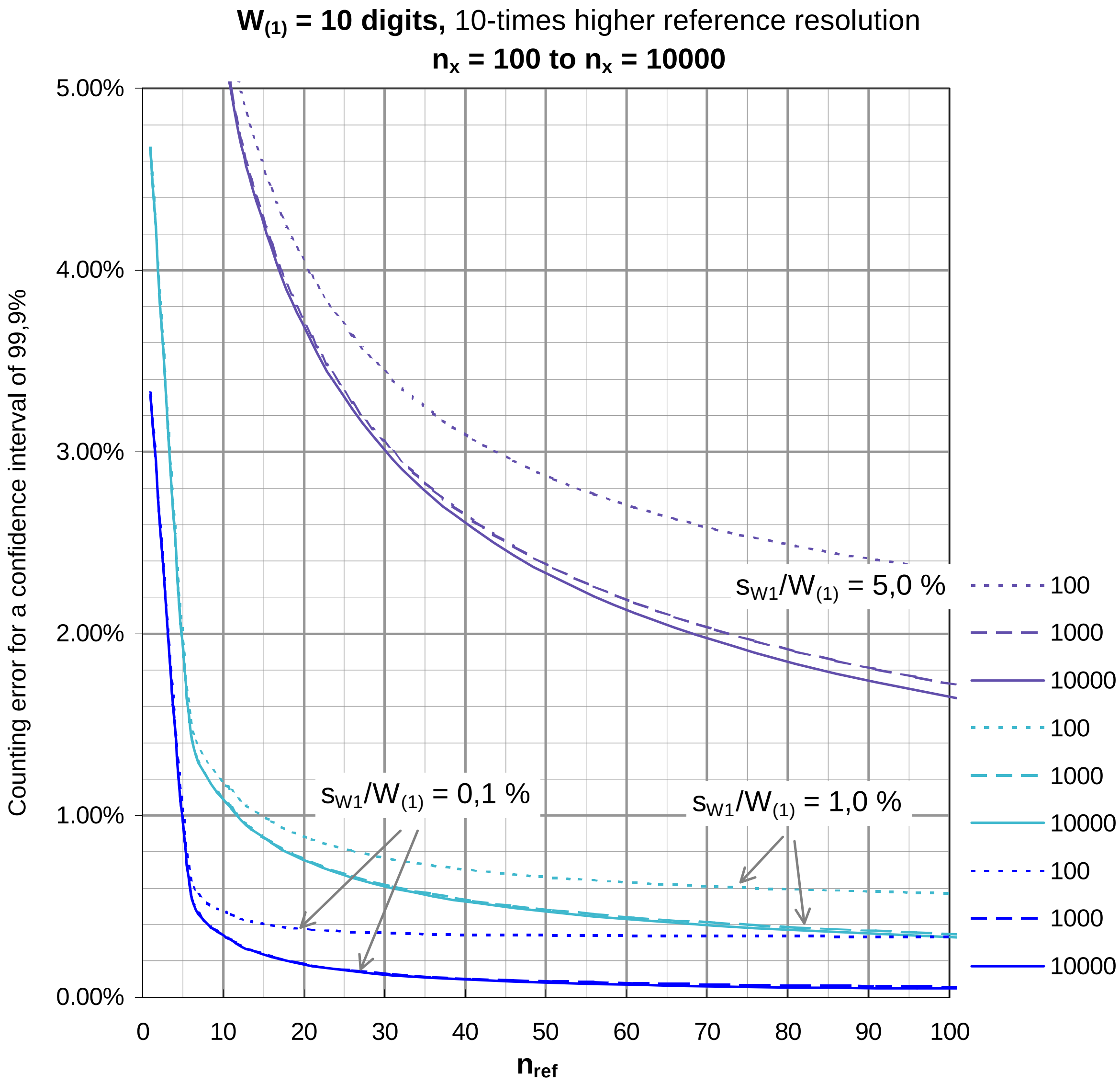


Figure 8: Relative counting error plotted as a function of the reference piece count for various target piece counts n_x

The graph shows that the smaller the total number of parts to be counted is, the larger the relative counting error becomes. For example, for a reference piece count of 80 and a relative standard deviation of 1.0%, the counting error equals

- 0.37 % for 10,000 parts (corresponds to ± 37 parts)
- 0.38 % for 1,000 parts (corresponds to ± 4 parts)
- and 0.59 % for 100 parts (corresponds to ± 1 part).

For a reference piece count of 80 and a relative standard deviation of 0.1 %, the counting error is equal to

- 0.10 % for 10,000 parts (corresponds to ± 10 parts)
- 0.10 % for 1,000 parts (corresponds to ± 1 part)
- and 0.34 % for 100 parts, which corresponds to ± 0.3 part and means that counting that is accurate down to the last component is possible under these conditions.

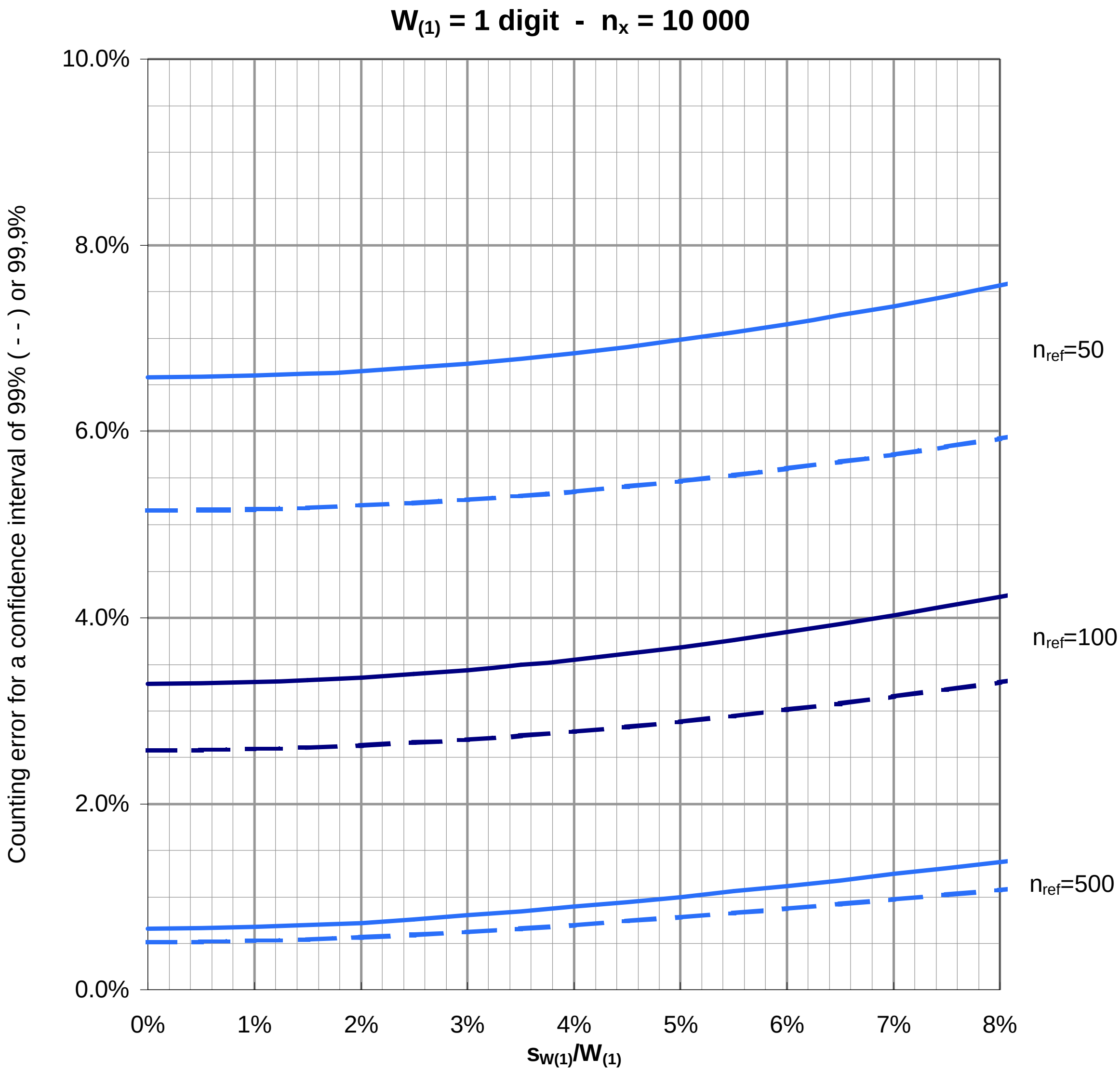


Figure 9: Relative counting error plotted as a function of the individual piece variance for various reference piece counts and an individual average piece weight of 1 digit
reference resolution = total piece count resolution.

When the relative standard deviation of the individual parts $\frac{s_{W_{(1)}}}{W_{(1)}}$ is known in %, the attainable counting error can be read off the y axis for reference piece counts of 50, 100 or 500.

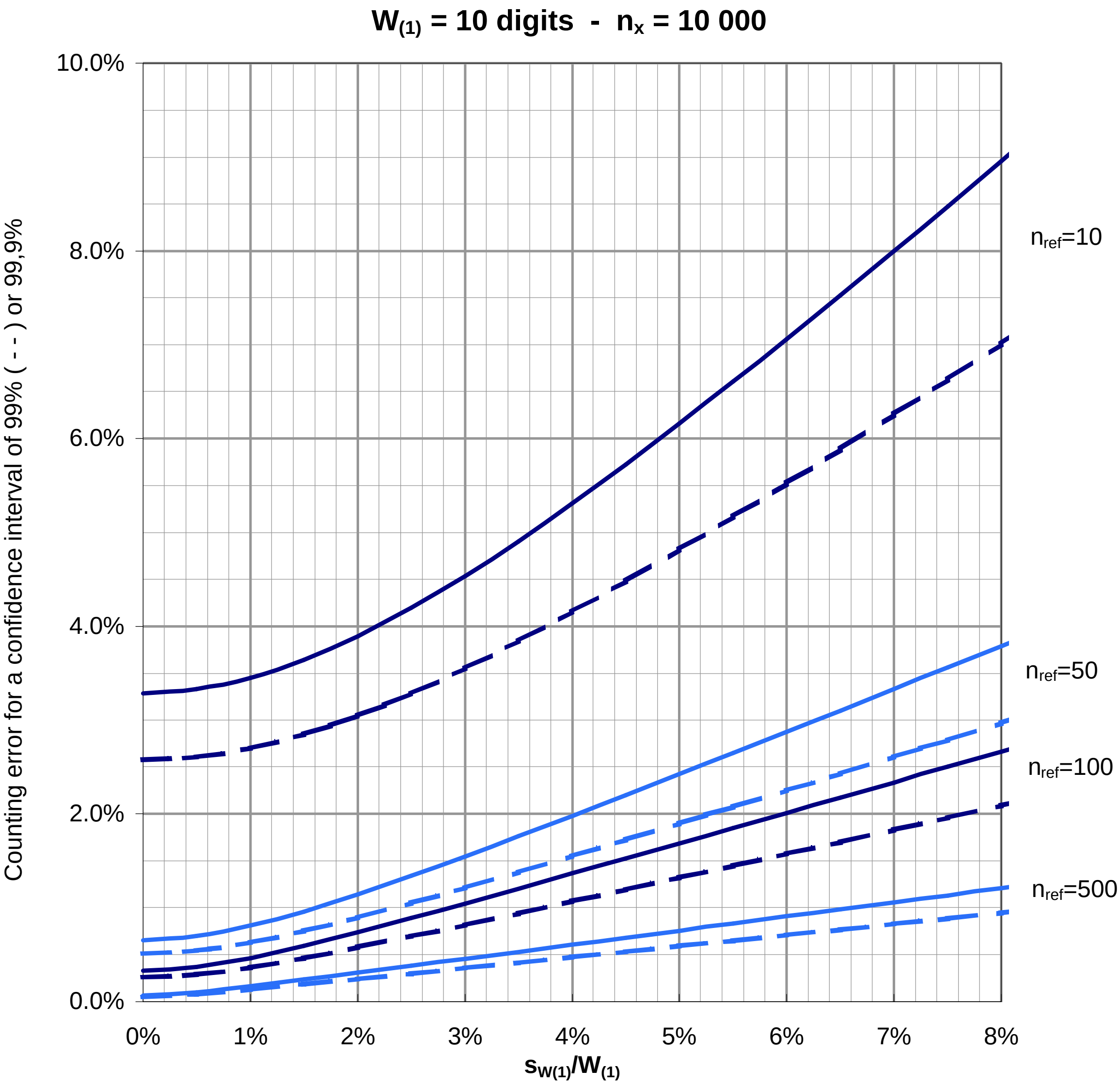


Figure 10: Relative counting error plotted as a function of the individual piece variance for various reference piece counts and an individual average piece weight of 10 digits
reference resolution = total piece count resolution.

When the relative standard deviation of the individual parts $\frac{s_{W_{(1)}}}{W_{(1)}}$ is known in %, the attainable counting error can be read off the y axis for reference piece counts of 10, 50, 100 or 500.

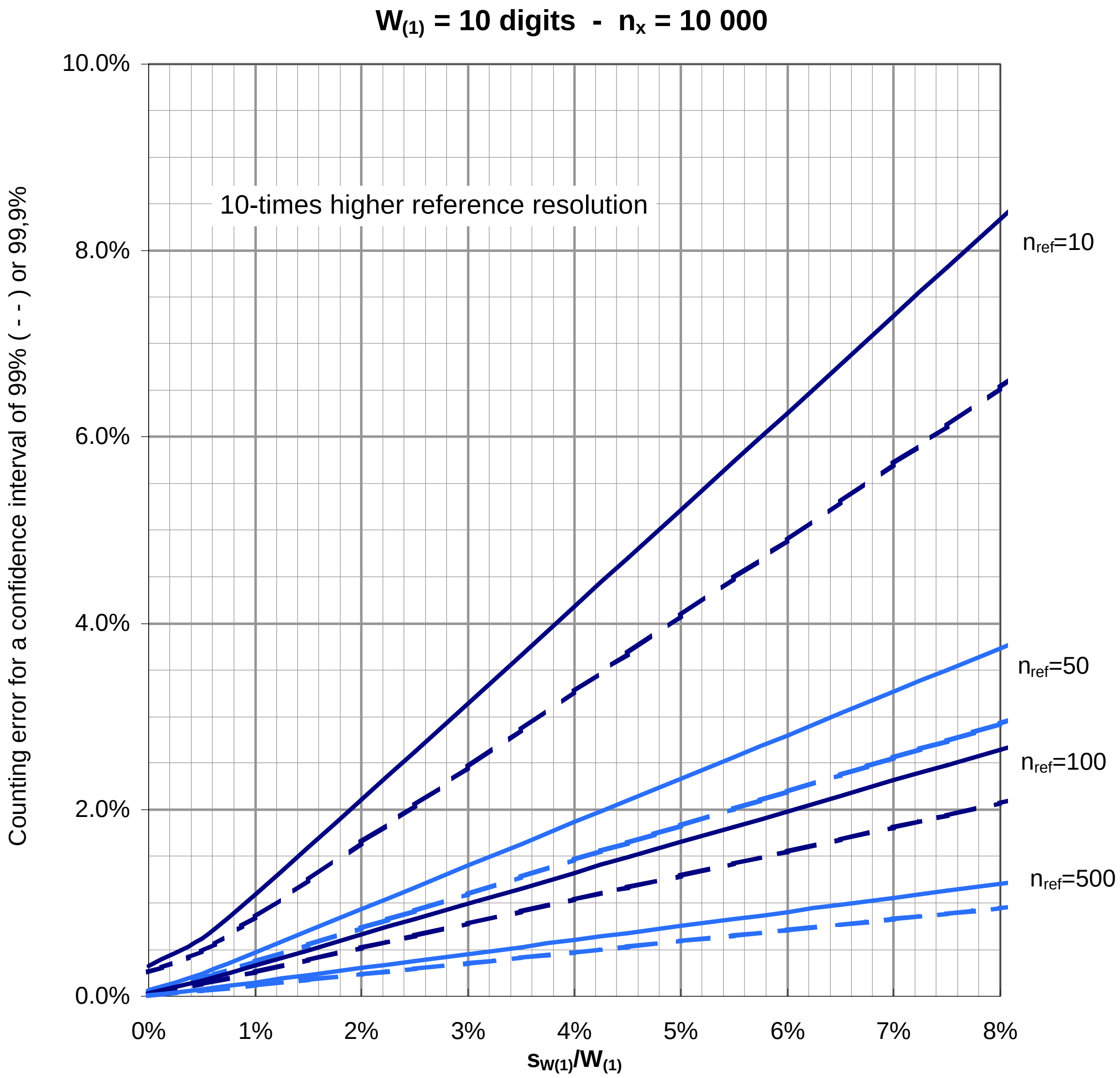


Figure 11: Relative counting error plotted as a function of the individual piece variance for various reference piece counts and an individual average piece weight of 10 digits
reference resolution = 10 · total piece count resolution.

When the relative standard deviation of the individual parts $\frac{s_{W_{(1)}}}{W_{(1)}}$ is known in %, the attainable counting error can be read off the y axis for reference piece counts of 10, 50, 100 or 500.

The advantage of higher reference resolution is especially noticeable when the values of the individual piece variance are low. When the individual piece variance is great, the counting accuracy is limited by this “disproportion of the parts.”

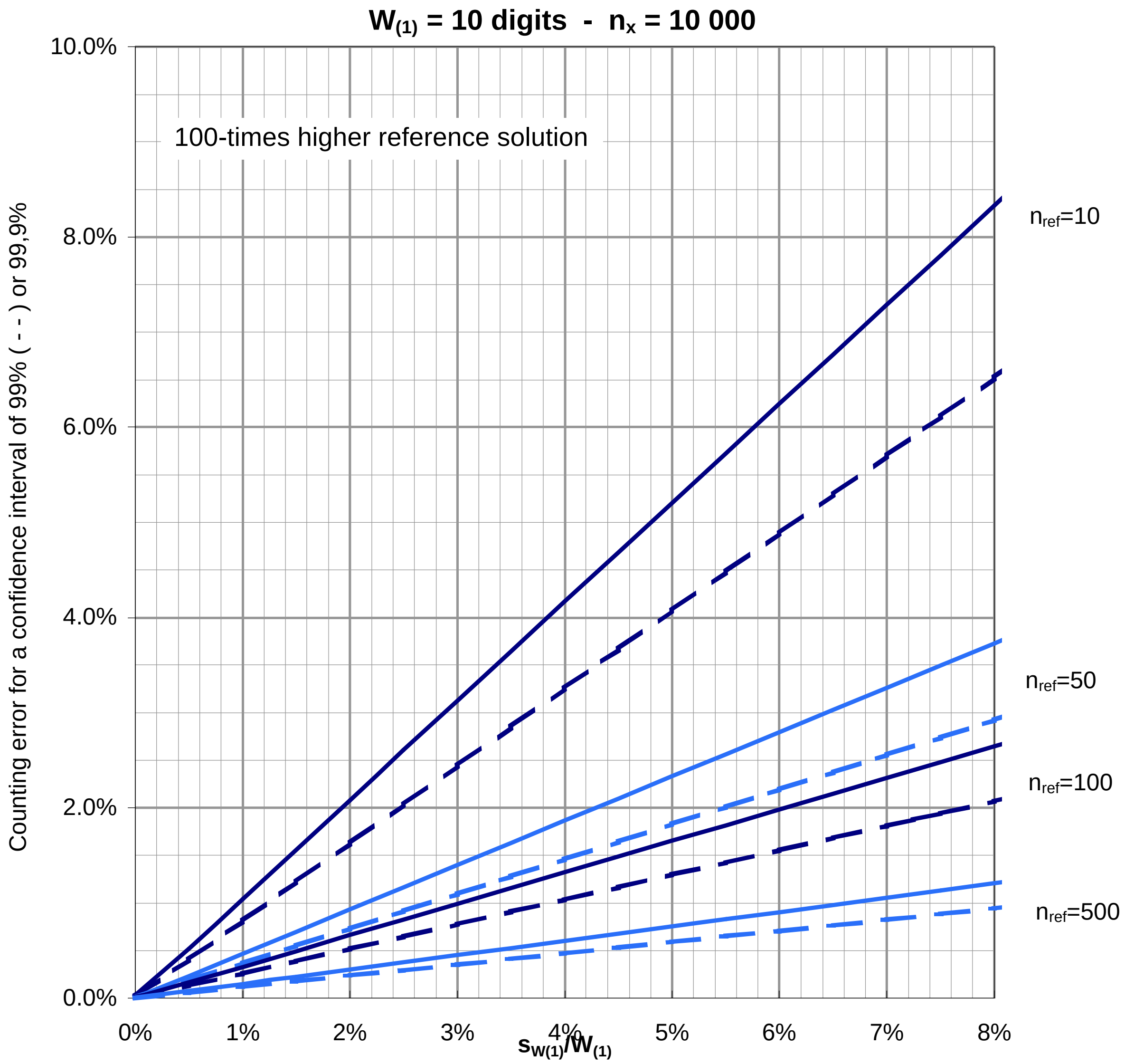


Figure 12: Relative counting error plotted as a function of the individual piece variance for various reference piece counts and an individual average piece weight of 10 digits
reference resolution = 100 · total piece count resolution.

When the relative standard deviation of the individual parts $\frac{s_{W_{(1)}}}{W_{(1)}}$ is known in %, the attainable counting error can be read off the y axis for reference piece counts of 10, 50, 100 or 500.

The advantage of higher reference resolution is especially noticeable when the values of the individual piece variance are low (see the previous graph). When the individual piece variance is great, the counting accuracy is limited by the weight variance of the individual parts.

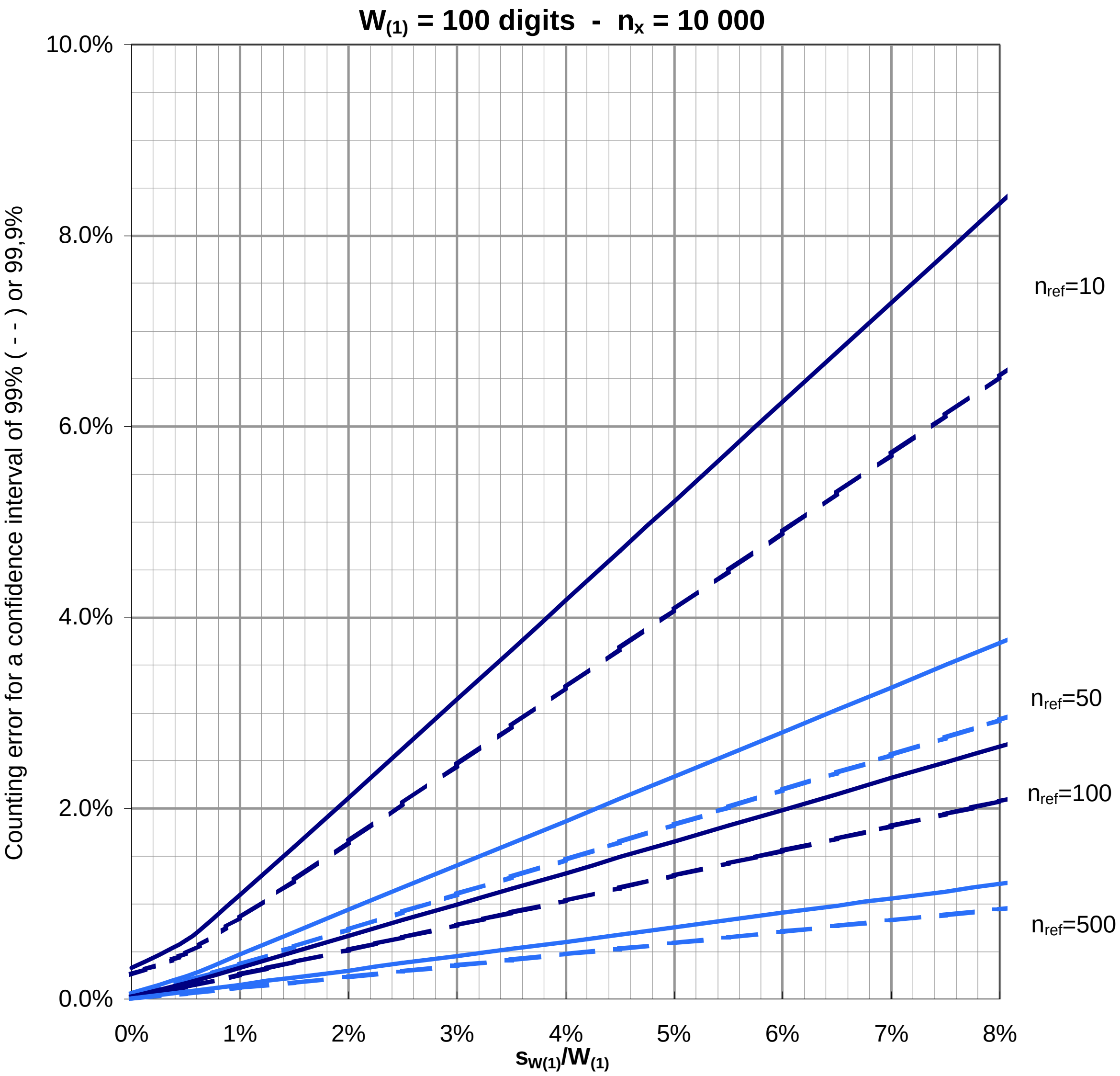


Figure 13: Relative counting error plotted as a function of the individual piece variance for various reference piece counts and an individual average piece weight of 100 digits
reference resolution = total piece count resolution.

When the relative standard deviation of the individual parts $\frac{s_{W_{(1)}}}{W_{(1)}}$ is known in %, the attainable counting error can be read off the y axis for reference piece counts of 10, 50, 100 or 500. The results are equal to those that also would be obtained using a scale with a readability corresponding to 10 digits of the individual average piece weight at 10 times higher reference resolution.

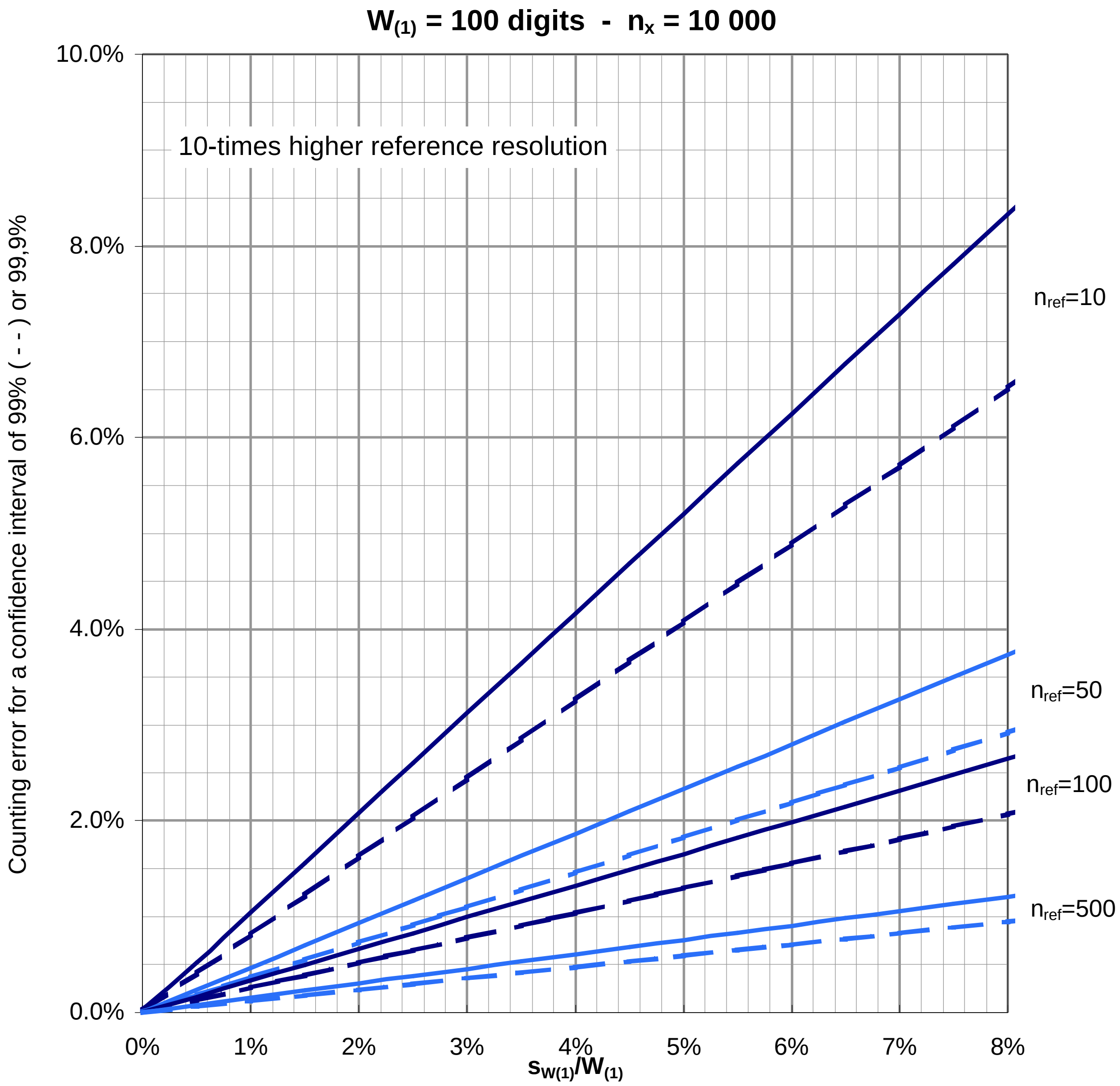
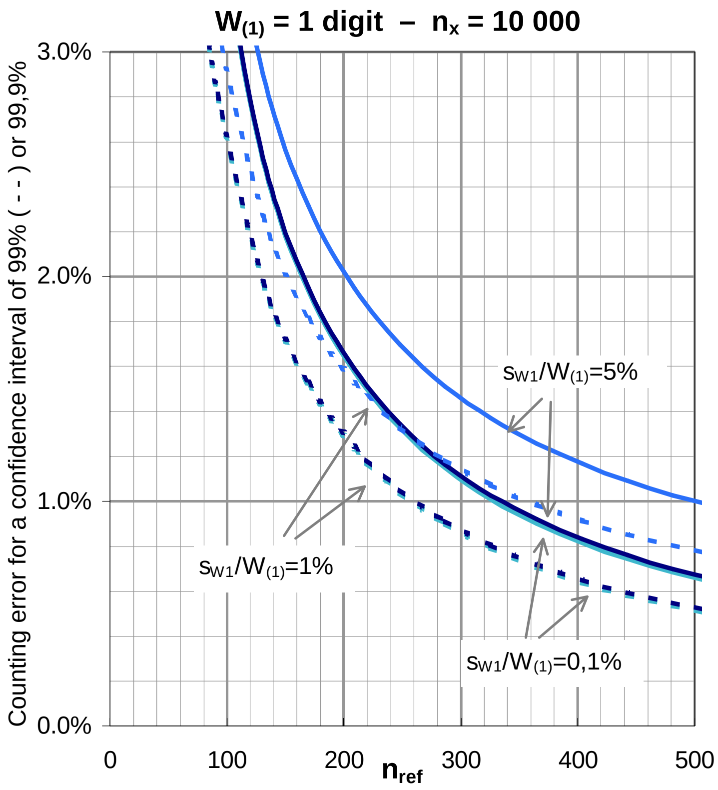


Figure 14: Relative counting error plotted as a function of the individual piece variance for various reference piece counts and an individual average piece weight of 100 digits
reference resolution = 10 · total piece count resolution.

When the relative standard deviation of the individual parts $\frac{s_{W_{(1)}}}{W_{(1)}}$ is known in %, the attainable counting error can be read off the y axis for reference piece counts of 10, 50, 100 or 500.

The advantage of higher reference resolution is especially noticeable when the values of the individual piece variance are low (see the previous graph). When the individual piece variance is great, the counting accuracy is limited by the variance of the average weight of the individual parts.



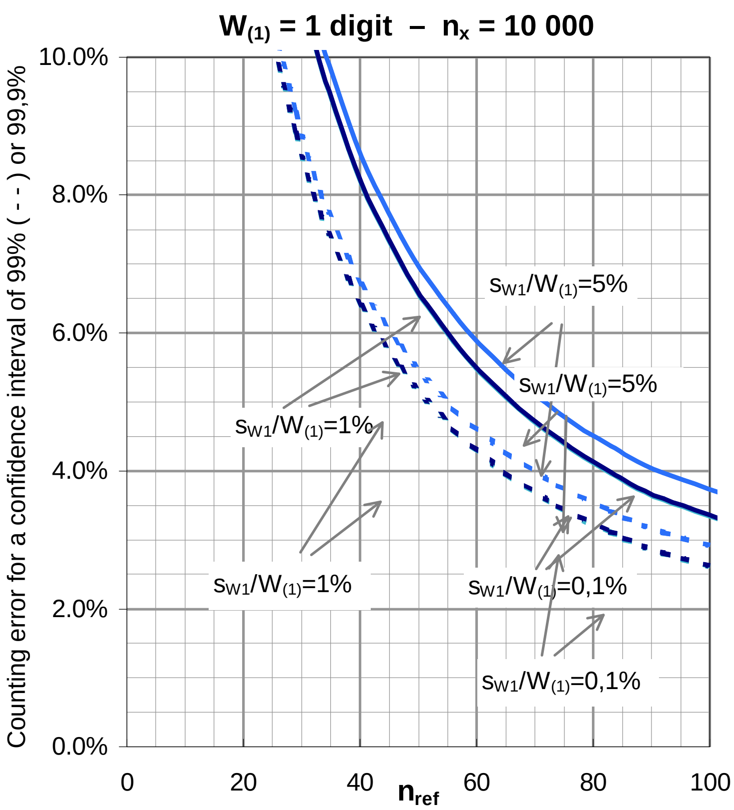


Figure 15: Relative counting error plotted as a function of the reference piece count for three different values of the individual piece variance and an individual average piece weight of 1 digit.

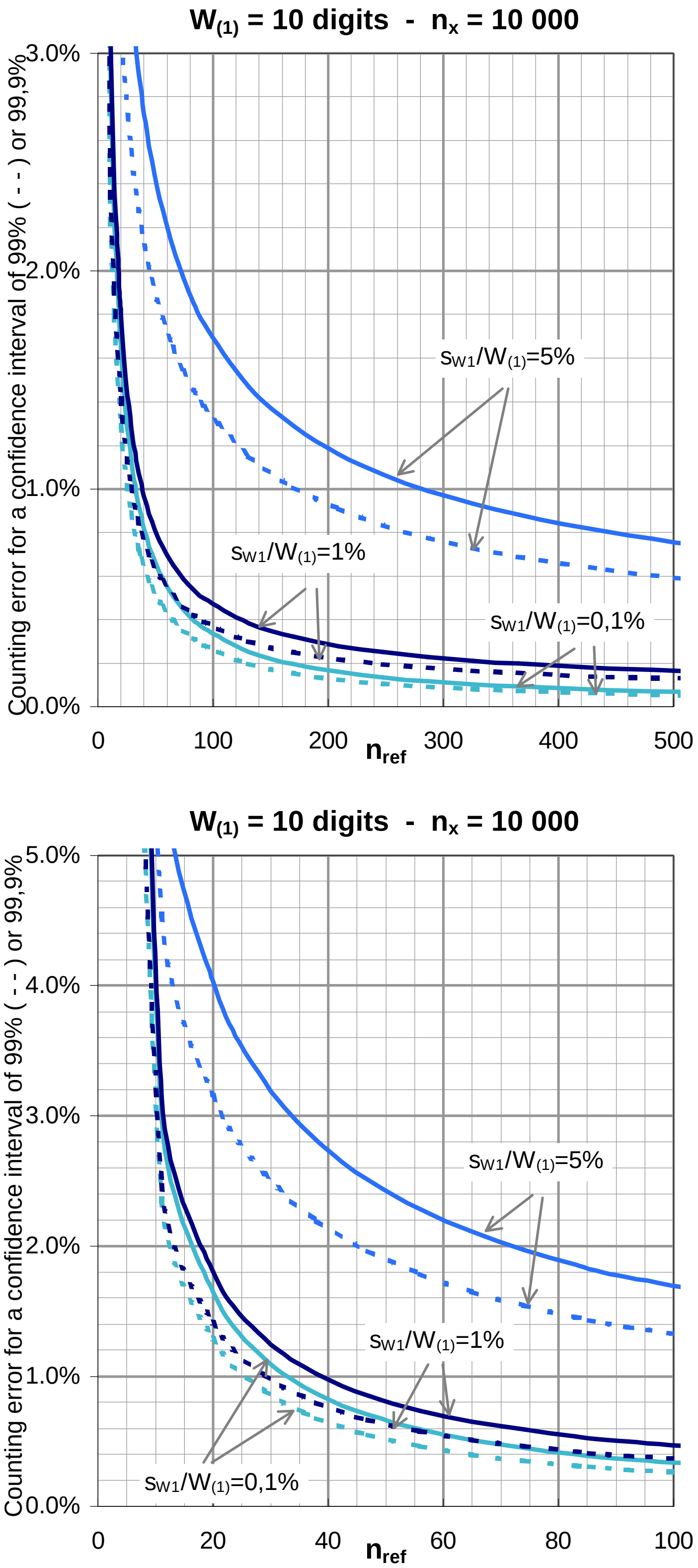
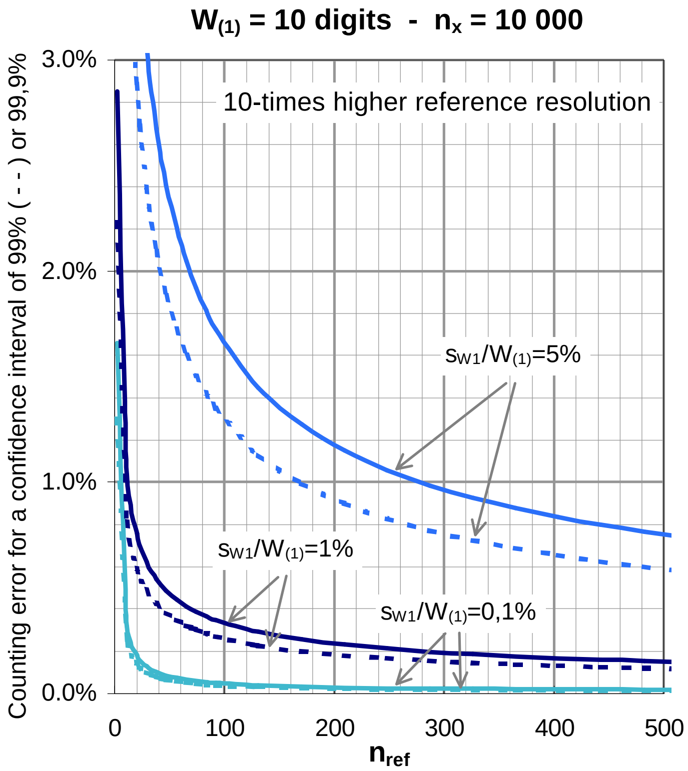


Figure 16: Relative counting error plotted as a function of the reference piece count for three different values of the individual piece variance and an individual average piece weight of 10 digits.



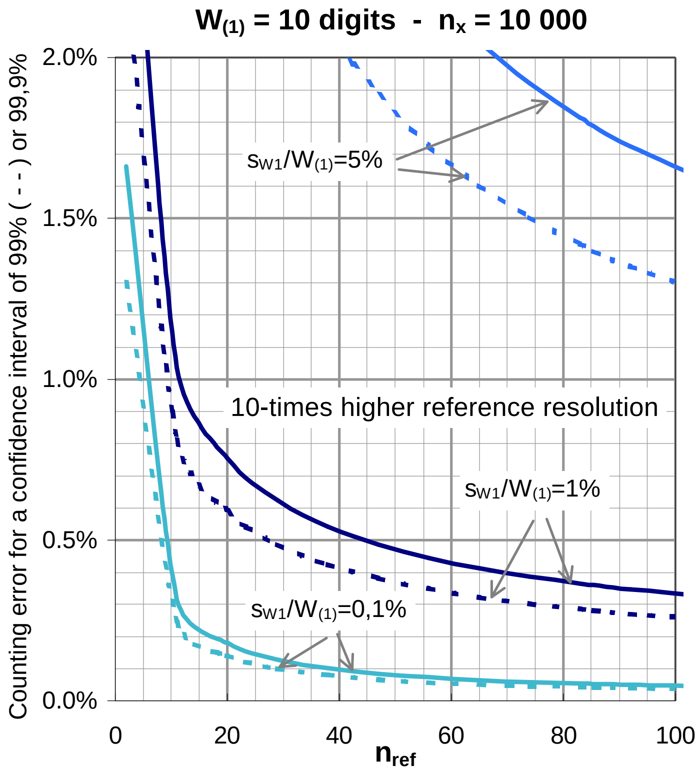
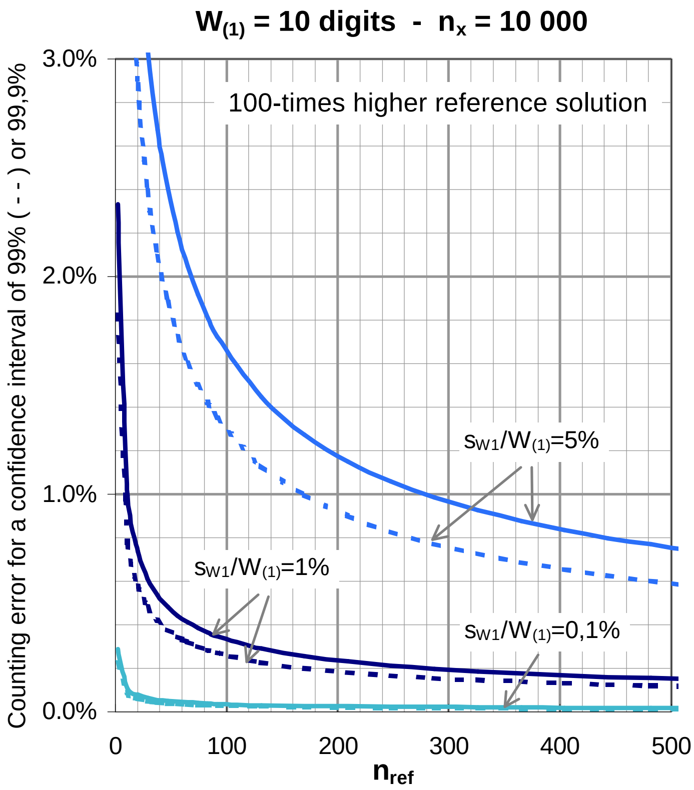


Figure 17: Relative counting error plotted as a function of the reference piece count for three different values of the individual piece variance and an individual average piece weight of 10 digits.



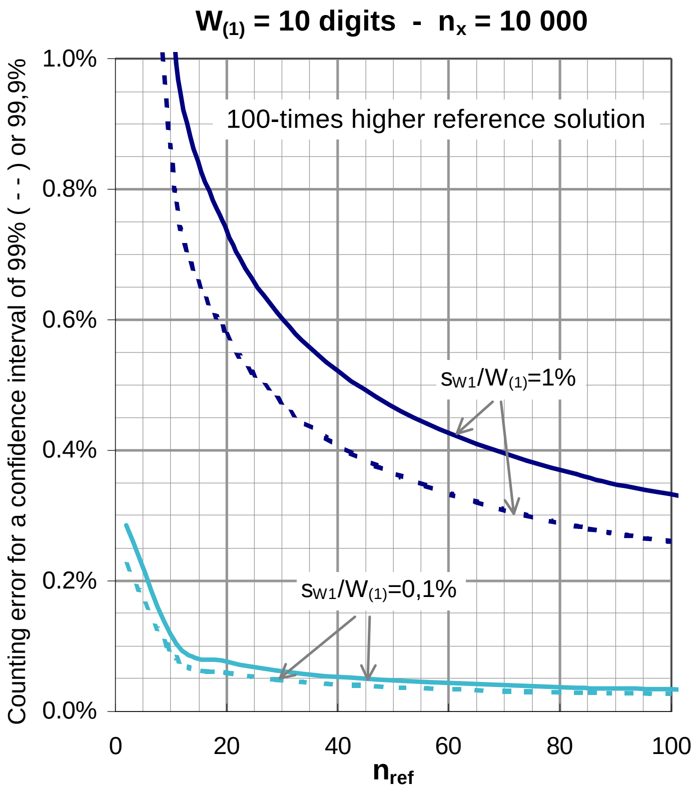


Figure 18: Relative counting error plotted as a function of the reference piece count for three different values of the individual piece variance and an individual average piece weight of 100 digits.

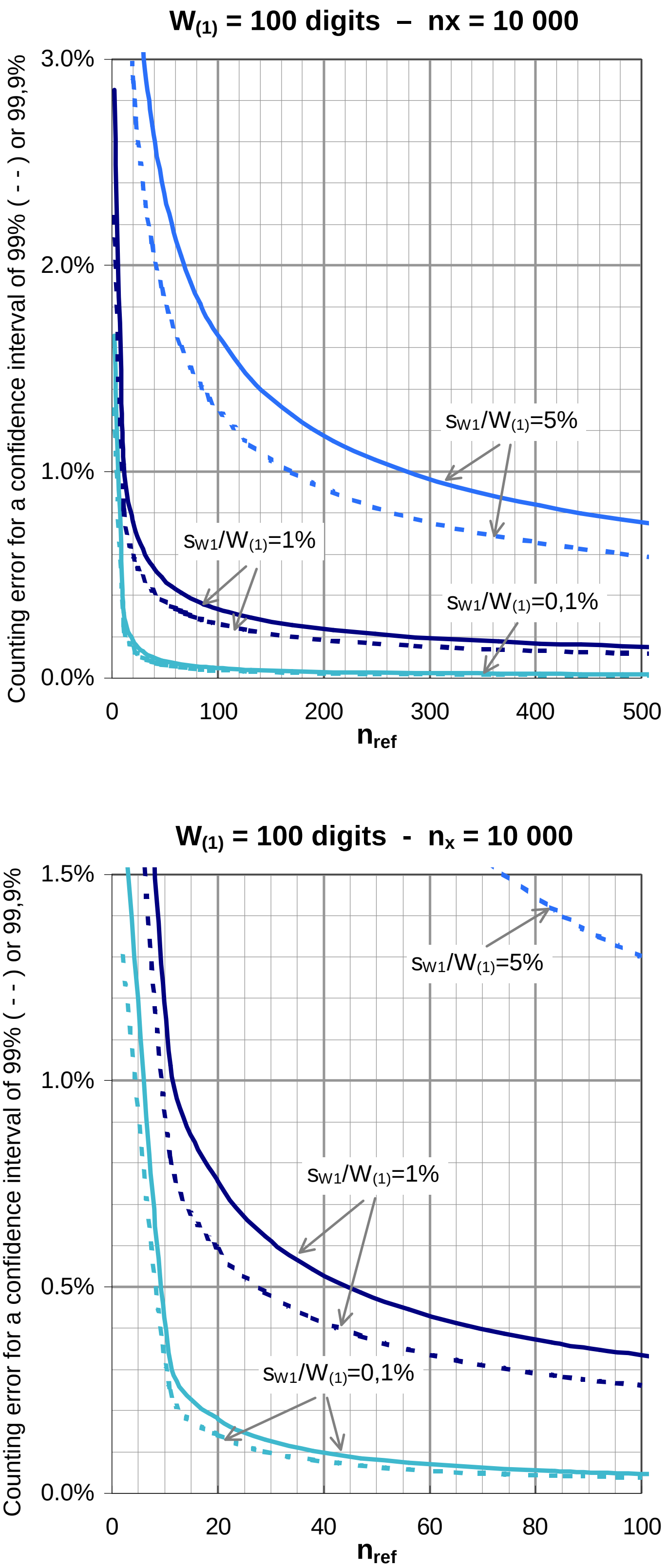


Figure 19: Relative counting error plotted as a function of the reference piece count for three different values of the individual piece variance and an individual average piece weight of 100 digits.

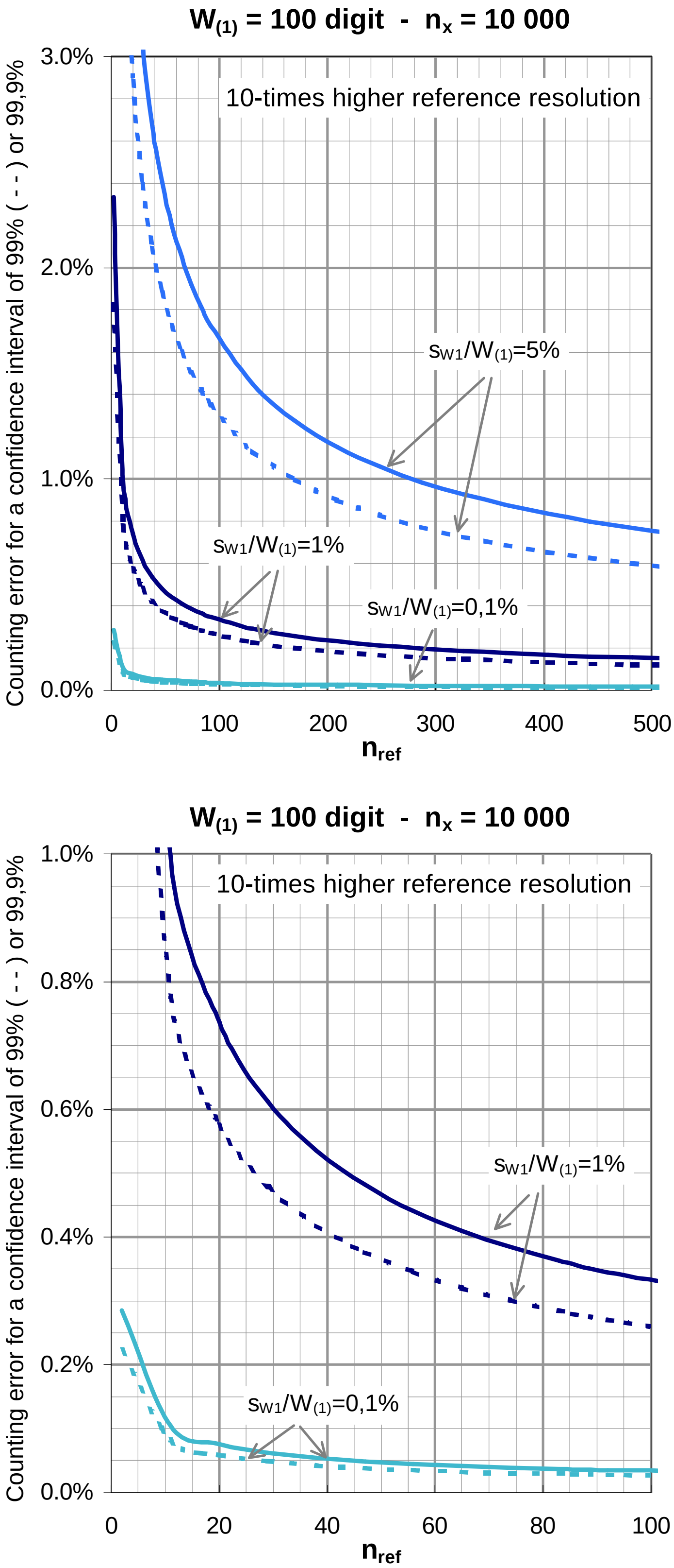


Figure 20: Relative counting error plotted as a function of the reference piece count for three different values of the individual piece variance and an individual average piece weight of 100 digits.

Calculating the Counting Accuracy – Examples of Printouts from the EXCEL File

With the help of the EXCEL file entitled "ACCURACY.XLS," the counting error can easily be calculated for every counting task. The individual weights of at least 6, preferably 10, carefully selected, representative individual parts are determined and entered in the fields highlighted in green.

The mean value and the standard deviation are calculated and shown in the yellow fields as essential results. (Additionally, the minimum and maximum weights and the standard deviation are shown in the gray fields.)

Entries must be made in the remaining green fields. The following information is required:

- the (approximate) number of individual parts to be counted,
- the reference piece count,
- the readability of the scale
- the readability of the scale used to determine the reference or the reference scale.

The formulas discussed in the first two chapters are used for calculation.

The standard deviation of the counting result is then indicated as an absolute value and as a percentage in the result field.

In addition, the maximum deviation from the target piece count and the relative counting error are indicated for the statistical probabilities of 95 %, 99 % and 99.9 %.

Examples of printouts for various counting conditions appear on the following pages.

Determining the Counting Accuracy

The counting accuracy is mainly influenced by the variance of the weight of the individual parts.

Therefore, the weights of 10 **different** individual parts must first be determined.

The smallest piece weight should amount to at least 10 digits of the scale display - 100 digits when a higher degree of accuracy is required.

Part	Indiv. Weight	g
1	2.0020	g
2	2.0000	g
3	2.0000	g
4	2.0010	g
5	2.0000	g
6	2.0000	g
7	1.9980	g
8	1.9990	g
9	2.0000	g
10	2.0010	g

Statistical Evaluation		
Largest individual value	2.0020	g
Smallest individual value	1.9980	g
Maximum deviation (largest value minus smallest value)	0.0040	g
Avg. weight of individual parts	**2.0001**	**g**
Standard deviation	0.0011	g
Relative standard deviation	**0.06**	**%**

Information Necessary for Calculating the Counting Accuracy

Number of parts to be counted (target piece count)	**1002**	
Reference piece count	**10**	
1 digit of the scale used	**0.1**	**g**
1 digit of the (reference) scale used or of the internal resolution	**0.01**	**g**
Total weight	**2004**	**g**

Result of the Counting Accuracy

Standard deviation of the piece count	**0.533**	
Relative standard deviation	**0.05**	**%**

		Statistical Probability		
		95%	99%	99.9%
Maximum deviation from target piece count	±	**1**	**1**	**2**
Relative counting error	±	**0.10%**	**0.14%**	**0.18%**
Counting accurate to last component possible		no	no	no
Fulfills the condition of the FPVO		yes	yes	yes

Determining the Counting Accuracy

The counting accuracy is mainly influenced by the variance of the weight of the individual parts.

Therefore, the weights of 10 **different** individual parts must first be determined.

The smallest piece weight should amount to at least 10 digits of the scale display - 100 digits when a higher degree of accuracy is required.

Part	Indiv. Weight	mg
1	23.1000	mg
2	23.2000	mg
3	23.3000	mg
4	23.2000	mg
5	23.1000	mg
6	23.2000	mg
7	23.1000	mg
8	23.2000	mg
9	23.0000	mg
10	23.2000	mg

Statistical Evaluation		
Largest individual value	23.3000	mg
Smallest individual value	23.0000	mg
Maximum deviation (largest value minus smallest value)	0.3000	mg
Avg. weight of individual parts	**23.1600**	**mg**
Standard deviation	0.0843	mg
Relative standard deviation	**0.36**	**%**

Information Necessary for Calculating the Counting Accuracy

Number of parts to be counted (target piece count)	**300**	
Reference piece count	**10**	
1 digit of the scale used	**0.1**	**mg**
1 digit of the (reference) scale used or of the internal resolution	**0.01**	**mg**
Total weight	**6948**	**mg**

Result of the Counting Accuracy

Standard deviation of the piece count	**0.351**	
Relative standard deviation	**0.12**	**%**

		Statistical Probability		
		95%	99%	99.9%
Maximum deviation from target piece count	±	**1**	**1**	**1**
Relative counting error	±	**0.23%**	**0.30%**	**0.39%**
Counting accurate to last component possible		no	no	no
Fulfills the condition of the FPVO		yes	yes	yes

Questions about Counting

1. On which fundamental equation is “counting by weighing” based?
2. Which quantities influence the counting accuracy, and which quantity has the greatest influence?
3. How does the reference piece count influence the accuracy of the counting result?
4. What is the advantage of reference optimization? Within what range should the increase in the piece count lie?
5. What is meant by “representative sampling,” and why is sampling while selecting the reference samples of such great significance?
6. Can you determine the standard deviation for the following series of individual weights?
   11.10 g / 10.98 g / 10.96 g / 10.99 g / 11.02 g / 11.06 g / 11.02 g / 11.00 g / 11.03 g / 10.92 g
7. If you calculated the standard deviation “by hand,” you already know the mean value of the weights above. In case you simply used the applicable EXCEL function, please use the function at this time to calculate the mean value. How great is the relative standard deviation?
8. Which scale would you recommend to someone who would like to count the parts in the example in question 6 accurately down to the last component in groups of 800?

Answers to the Questions

1. $n_x = W_x \cdot \frac{n_{ref}}{W_{ref}}$

2. The variance of the average individual piece weight has the greatest influence on the counting accuracy.

   Other influence quantities: reference piece count, total piece count, variance of the reference weight, variance of the total weight

3. In general, the larger the reference piece count is, the greater becomes the degree of counting accuracy.

4. For large reference piece counts, it is not necessary to count all individual parts by hand, rather only the first 10 or 15 parts, for example. This reduces the probability of a counting error occurring during determination of the reference piece count.

   During reference optimization, the reference piece count should be increased by at least 2 parts. A maximum of twice the original reference piece count may be used.

5. For a description of sampling, see p. 24.

   Based on the properties (here the weight) of a few parts, an inference is drawn regarding the properties of a large number of parts – with the help of statistical methods. If the reference samples do not represent the population of all parts, the basic requirement for the applicability of the statistical calculations is missing.

6. The value is calculated according to the general formula (equation 6, Fundamentals of Statistics)

   $$s = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2} = \pm\, 0{,}051\text{g}$$

   In EXCEL, this corresponds to the STDEV(...) function, <u>not</u> the STDEVP(…) function.

7. The mean value $\bar{x}$ is 11.01 g. (corresponds to the EXCEL function AVERAGE(…).)

   The relative standard deviation is $\frac{s}{\bar{x}} = 0{,}0046 = 0{,}46\%$.

8. Table 5 on p. 23 shows that it is impossible to count 1,000 parts accurately down to the last component when the relative standard deviation of the individual parts is 0.5 %. Table 4 on p. 22, shows that an error of ± 0.1 % or ± 1 related to 1,000 parts is attainable (e.g., with a scale readability of 1 g and 100-times higher internal resolution or a scale readability of 0.1 g and 10-times higher internal resolution and a reference piece count of 279 (use the reference optimization function).

   If you use the Excel file entitled "ACCURACY.XLS," you will see that under the given conditions, it is possible to count the parts accurately down to the last component with a reference piece count of 290 and a confidence interval of 95 %. Counting with a confidence interval of 99 % or even 99.9 %, however, is not possible. This means that the quantity weighed is really 800 parts in 95 out of 100 counting operations. In 5 out of 100 cases, the actual piece count on the scale can be 801 or 799 when the display shows 800.

Register